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A MODIFIED POSICAST METHOD OF CONTROL WITH APPLICATIONS TO HIGHER ORDER SYSTEMS

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THESIS

A MODIFIED POSICAST METHOD OF CONTROL

WITH

APPLICATIONS TO HIGHER ORDER SYSTEMS

by

Hyo Chung So

Lieutenant, ROK Navy

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Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE

IN

ELECTRICAL ENGINEERING

United States Naval Postgraduate School
Monterey, California

1960

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Hyo Chung So

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The control mechanism of the transistor control is completely known. A step command is given to the system. In general, the second part must follow the first part by nearly one-half cycle of the transient.

The total transient time is limited to about one-half cycle. There is no transient overshoot and no oscillation.

The first command must deliver exactly the amount of energy needed to reach the desired output displacement. The second step and additional compensation circuit and switching or signal removes all the stored energy and locks the system.

The writer wishes to express his appreciation for the assistance and encouragement given him by Professor George J. Thaler in this project.

TABLE OF CONTENTS

	Title	Page
Introduction		1
Section 1.	Theory of Second Order System	2
Section 2.	Third Order System	11
2.1	Theory of Third Order System	11
2.2	Computer Studies	16
2.3	Phase Space Technique	27
Section 3.	Fourth Order System	37
3.1	Theory of Fourth Order System	37
3.2	Evaluation of the System Transfer Function	38
3.3	Computer Studies	48
3.4	Physical System Studies	67
Discussion and Conclusions		78
Bibliography		80

LIST OF ILLUSTRATIONS

Figure		Page
1.	Typical Transient Response of a Second Order System With Posicast Control	3
2.	Cascade and Feedback Compensation Schemes for a Second Order System	
	a) Cascade Compensator	5
	b) Feedback Compensator	6
	c) Two Delayed Steps with Feedback Compensator	10
3.	Typical Transient Response of a Second Order System with Posicast Control Using Two Delayed Steps	7
4.	Root Locus of a Third Order System	13
5.	Circuit for a Third Order Servo	
	a) Series Compensation, Acceleration Boundary Condition Shorted Out	17
	b) Series Compensation, Boundary Condition Shorted Out	21
	c) Series Compensation, Stored Energy Cancelled By Applied Pulse	22
6.	Block Diagrams for a Third Order System	18
7.	Posicast Control of a Third Order System Using Techniques of Fig. 6.	
	a) Transient Response Curves	
	1) Underdamped 2) Overdamped 3) With Posicast and Overdamping	24
	b) θ_c vs $\dot{\theta}_c$ Phase Plane	25
	c) θ_c vs $\ddot{\theta}_c$ Phase Plane	26
8.	Acceleration vs Velocity Trajectories with Damping Connected and Error Signal Cancelled at $N = -\omega_D$	29
9.	Computer Diagram for Third Order System Using the Phase Space Technique	30

10. Posicast Control of a Third Order System Using Phase Space Technique

a) θ_c vs $\dot{\theta}_c$ Phase Plane Family 35

b) Step Response (θ_c vs t) Family 36

11. Schematic Block Diagram of the Fourth Order Positioning Servo 39

12. Bode Diagram of the Fourth Order Positioning Servo 40

13. Schematic Block Diagram for the Open Loop Frequency Response Test of Amplidyne 41

14. Bode Diagram of Amplidyne Only 42

15. Pot. Calibration 43

16. Tachometer Calibration 44

17. Root Locus (Fourth Order System) 46

18. Block Diagram of Amplidyne Positioning Servo with Feedback Compensation 47

19. Computer Circuit and Results for Posicast Control of a Fourth Order System

A) Computer Circuit. Feedback Compensation, Boundary Conditions are Shorted Out 50

B) Results of Computer Studies 52

a) Underdamped

b) Overdamped

c) Final Result

d) Step Responses with Different Magnitudes of the Command

20. Computer Circuit and Results for Posicast Control of a Fourth Order System

A) Computer Circuit. Using Shorting Switches to Simulate Opening the Inductive Circuit 56

B) Result of Computer Studies 58

a) Underdamped

b) Posicast with Overdamping Circuit but Without Cancelling Stored Energy

c) Overdamped

d) Final Result with Either Short Circuit

21. Computer Circuit and Results for Posicast Control of a Fourth Order System

A) Computer Circuit. Using a Pulse Input to Counteract the Stored Energy 63

B) Final Results of Computer Studies with Pulse 65

C) Results of Computer Studies with Varying Magnitude of the Pulse and Pulse Width 66

22. Circuit Diagrams for Modified Posicast Control of Positioning Servo

a) With Open Circuit Switching to Dissipate Stored Energy 69

b) With Pulse Injection to Cancel Transient Due to Stored Energy 70

23. Response of the Physical System 71

a) Underdamped 71

b) Overdamped 72

24. Step Response of Servo with Modified Posicast Control. Using Circuit of Fig. 22a 73

1) Posicast, with Switching in Quadrature Field

2) Posicast without Cancellation of Stored Energy

25. Family of Step Responses Opening the Quadrature Field of Amplidyne. (I and II) 74

26. Step Responses of Servo with Modified Posicast Control. Using Circuit of Fig. 22b 76

27. Effect of Pulse Width (τ) on the Transient Response 77

The theory of posicast control permits deadbeat step response from a very lightly damped system by a delayed step or steps. The original development deals primarily with second order systems, proposes a delay line technique for application of the second step, notes that use of this technique leaves the basic system underdamped, then the processes use of the delay line in a feedback path to damp load disturbances and finally proposes use of a tapped delay line for higher order systems.

It has also proposed that a delayed step may be applied to any order linear underdamped system by using a switch operated on a constant delay basis.

The delay time and step magnitudes are determined experimentally. The resulting response is not deadbeat but retains most of the advantages of posicast control. The delay line techniques have disadvantages when applied to relatively slow control systems, and the timer proposal makes no provision to damp load disturbances.

This paper proposes two basic modifications to the posicast scheme. The first is that the switching operation which introduces the delayed step also introduce compensation circuits which overdamp the system without decreasing the static gain. These circuits do not affect the step response, but provide damping for load disturbances and noise without changing the stiffness of the system. The second is the application of the technique to higher order system with provision for handling the boundary condition problems at the switching instant by dissipating or counteracting the stored energy in the pertinent components. The basic theory, special techniques, including a phase space approach, and computer studies are detailed in the text, plus studies of an actual positioning servo with experimentally determined forth order transfer function.

SECTION 1 THEORY OF SECOND ORDER SYSTEM

This theory has been adequately covered in the literature and is only summarized here. Given a second order underdamped system with transfer function

$$\frac{\theta_c}{E}(S) = \frac{K}{S(S+P)} \tag{1}$$

the system function is

$$\frac{\theta_c}{\theta_R}(S) = \frac{K}{S(S+P)+K} = \frac{K}{(S+\alpha-j\omega_c)(S+\alpha+j\omega_c)} \tag{2}$$

and the transient response to a step input, A (t), is

$$\theta_c(t) = A + \frac{\omega_n A}{\omega_c} e^{-\alpha t} \sin(\omega_c t - \phi) \tag{3}$$

where $\omega_n = (\alpha^2 + \omega_c^2)^{\frac{1}{2}}$. The maximum overshoot occurs at $t = T_p = \pi/\omega_c$. $\phi = \tan^{-1} \omega_c/\alpha$. At this instant $\theta_c = 0$ and

$$\theta_c(T_p) = A + \frac{\omega_n A}{\omega_c} e^{-\frac{\alpha\pi}{\omega_c}} \sin\phi \triangleq aA \tag{4}$$

From this it follows that if the magnitude of the step is at A = 1/a, then at $t = T_p$

$$\theta_c(T_p) = 1.0 \quad (\text{for } A = 1/a) \tag{5}$$

It is apparent that at $t = T_p$, $\theta_c = 1$ and $\dot{\theta}_c = 0$ if a step $\theta_R = (1/a)u(t)$ was applied at t = 0. There still remains an error signal $E = \theta_R - \theta_c = (1/a) - 1$, but if this is removed by changing θ_R from 1/a to 1.0 at instant $t = T_p$, then the system is standstill at the desired position, there is no error signal to actuate the system, and (for the second order system) there is no stored energy to disturb the equilibrium. The transient response is therefore as shown on Fig. 1 (page).

The basic system is still underdamped and will oscillate if sub-

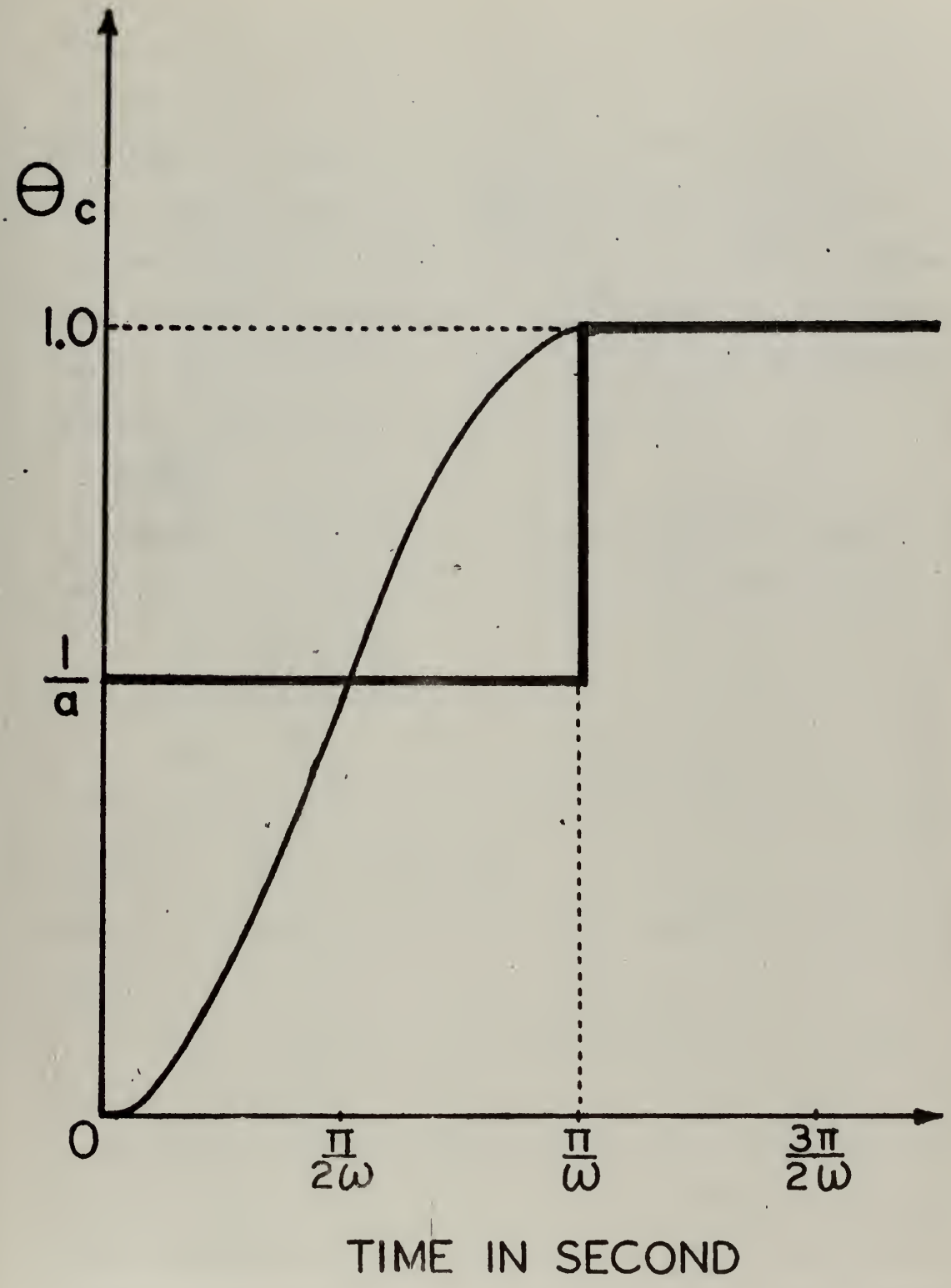


Fig. 1. Typical Transient Response of a Second Order System with Posicast Control.

are zero at the switching boundary (assuming no instrumentation noise disturbing the system, and there is no boundary layer problem). Fig. 2 (page 5) shows a scheme for introducing a pseudo coordinate, in which case switching may be accomplished with a relay, and Fig. 3 (page 6) shows a scheme for introducing tachometer feedback, in which case the $\dot{\theta}_c$ signal is available and may be used to actuate the switch when $\dot{\theta}_c = 0$. Note that high static gain is readily maintained.

EXAMPLE 1.

Consider a system with open loop transfer function such that the roots are on the imaginary axis. Then the characteristic equation is

$$S^2 + K = 0 \tag{1}$$

and for pure imaginary roots

$$K = \omega^2 \tag{2}$$

substitute K and $\omega_n = \omega_c = \omega$ into the equation 1. Then at $t = T_p = \pi/\omega$ and

$$\theta_c(T_p) = 2A = aA \tag{3}$$

Therefore $A = 1/a = 0.5$ and the transient response is shown on Fig. 4 (page 7).

In previous discussion, only one delayed step input is used. By using (more than one) two delayed step rise time is improved. In order to get deadbeat response to the unit step input and faster rise time, first step input may be greater than previous example, and connect second step which is smaller in magnitude or has polarity opposite to first step input before the system reaches desired response so that braking force is applied to the system and system response reaches unity as the velocity goes to zero. At the instant when $\dot{\theta}_c = 0$ and

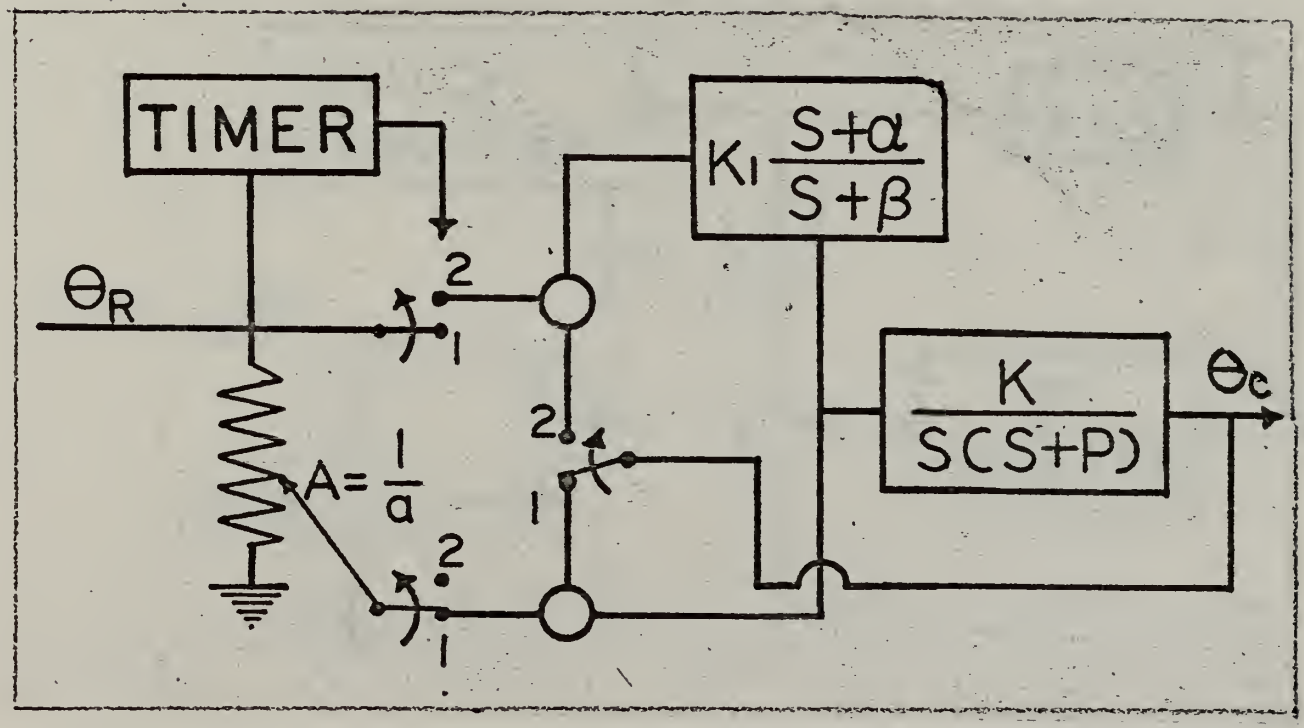
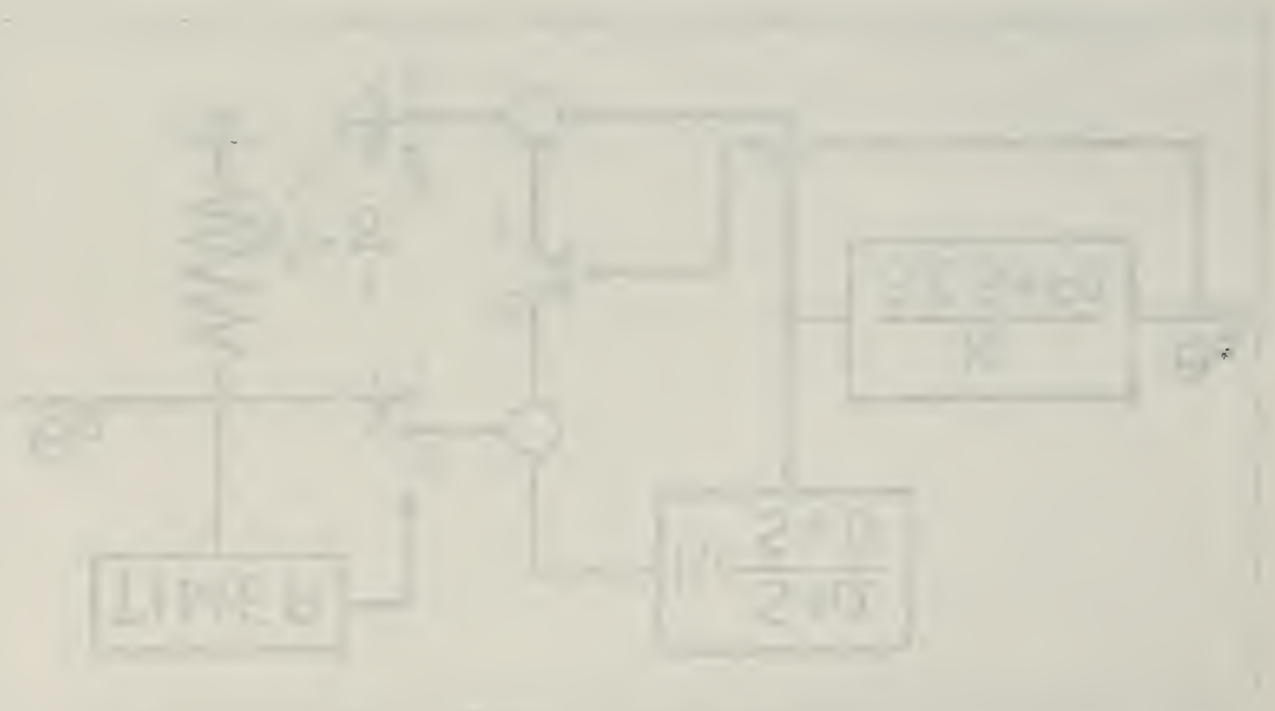


Fig. 2. Cascade and Feedback Compensation Schemes for a Second Order System.

a) Cascade Compensator.



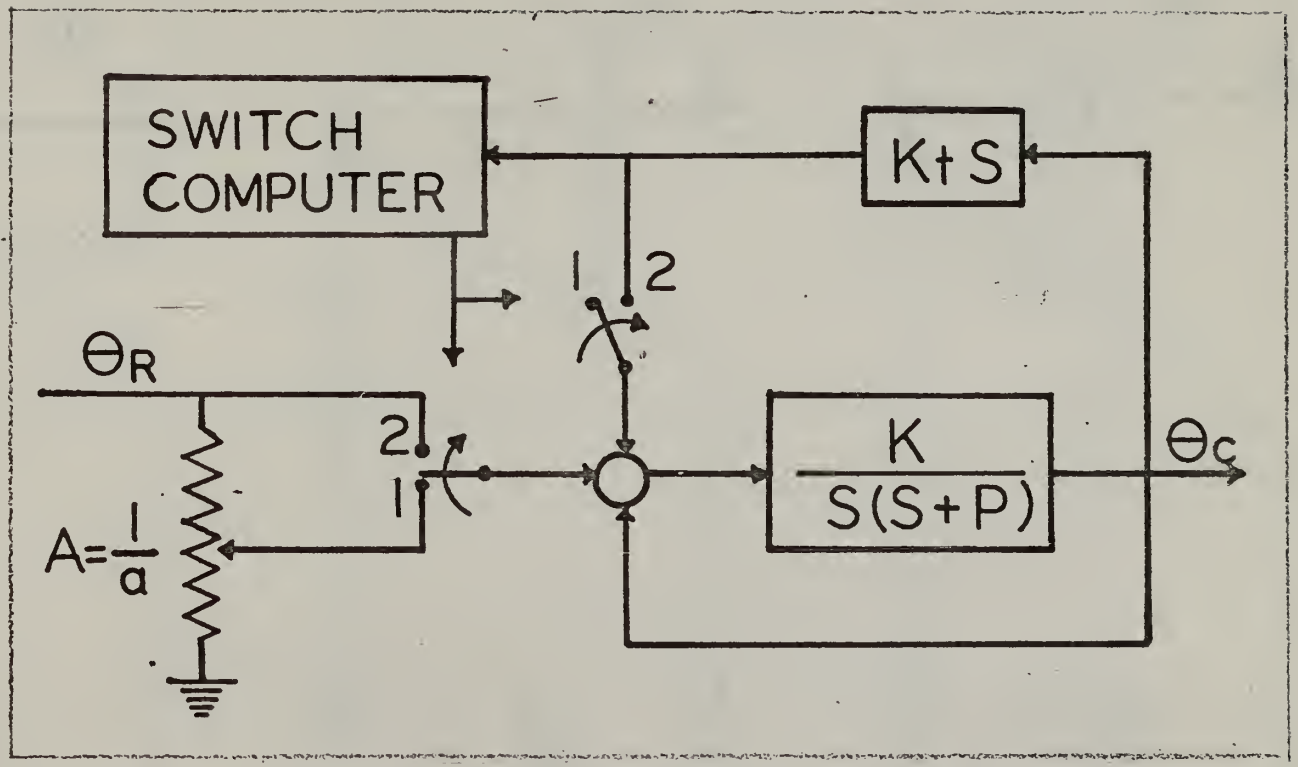


Fig. 2. Cascade and Feedback Compensation Schemes for a Second Order System.

b) Feedback Compensator.

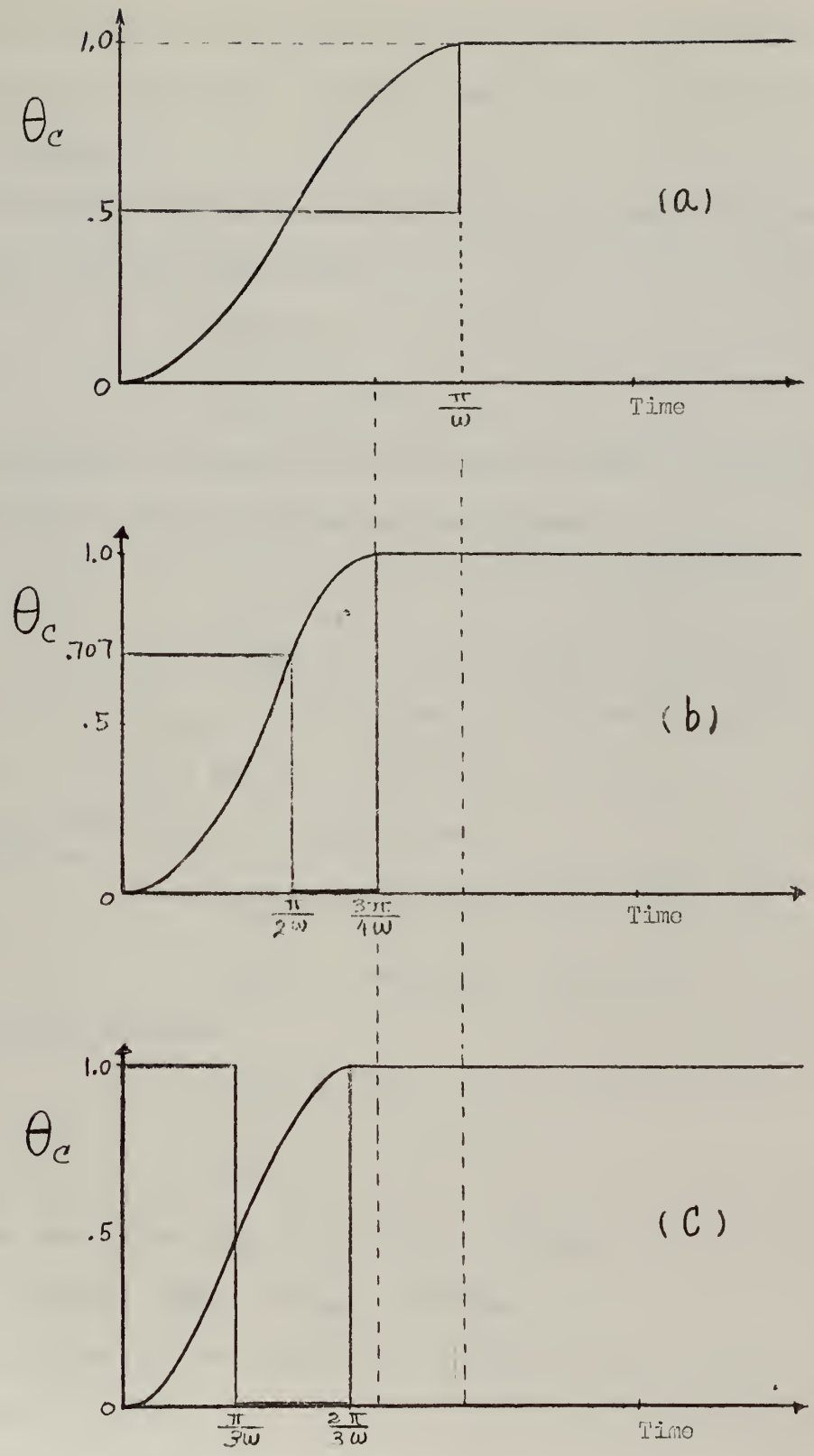
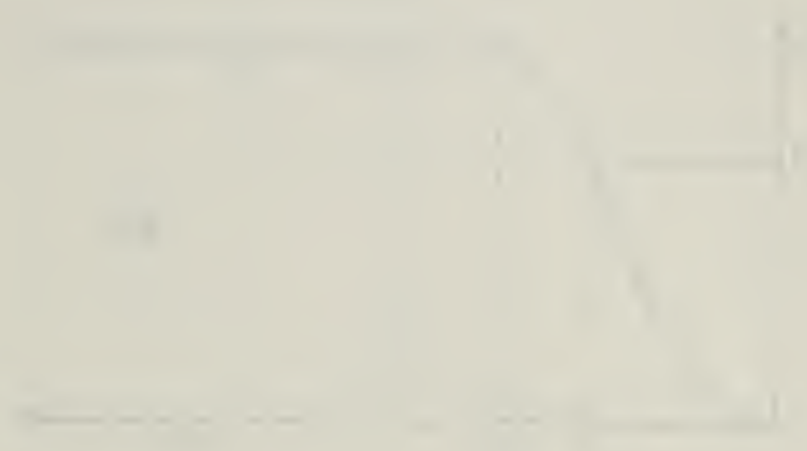


Fig. 3. Typical Transient Response of a Second Order System with Position Control Using Two Delayed Steps.



$\dot{\theta}_c = 0$, the unity step input and the overdamped compensation circuit are connected to the system. Typical examples are presented in the following.

EXAMPLE 2.

Using same system transfer function as in Example 1, the transient response to a step input $Au(t)$, is

$$\theta_{c1}(t) = A - A \cos \omega t$$

$$\dot{\theta}_{c1}(t) = A\omega \sin \omega t \qquad (9)$$

For convenience in applying to the physical system, the second step is chosen zero. Then the system response becomes

$$\theta_{c2}(t') = \theta_{ci} \cos \omega t' + \dot{\theta}_{ci}/\omega \sin \omega t' \qquad (10)$$

$$\dot{\theta}_{c2}(t') = - \theta_{ci}\, \omega \sin \omega t' + \dot{\theta}_{ci} \cos \omega t' \qquad (11)$$

where $t' = t + T_{p1}$. T_{p1} is the time when second step input is connected. And $\theta_{ci} = \theta_{c1}(T_{p1})$ and $\dot{\theta}_{ci} = \dot{\theta}_{c1}(T_{p1})$.

If $T_{p1} = \pi/2\omega$, $\theta_{ci} = A$, $\dot{\theta}_{ci} = A\omega$ and

$$\theta_{c2}(t') = A \cos \omega t' + A \sin \omega t'$$

$$\dot{\theta}_{c2}(t') = a\omega \sin \omega t' + a\omega \cos \omega t' \qquad (12)$$

For deadbeat response

$$\theta_{c2}(t') = 1$$

$$\dot{\theta}_{c2}(t') = 0 \qquad (13)$$

solving these Equations 13 and 14 for t' and A,

$$\omega t' = \pi/4\omega,\ 5\pi/4\omega,\ 9\pi/4\omega, \ldots \qquad (14)$$

Choosing shortest time required for $\dot{\theta}_{c2}(t') = 0$, $t' = t_{p2} = /4\omega$ and then $A = .707$.

The transient response is shown on Fig. 3b (page 7).

EXAMPLE 3.

Using same system and in the same manner as in Example 2.

If $t' = T_{p1} = \pi/3\omega$, then $\theta_{ci} = A/2$ and $\dot{\theta}_{ci} = (3/2)\,A\omega$.

And substitute in Equations 10 and 11 and solving Equation 14, $T_{p2} = /3\omega$ and first posicast step size A-1. The transient response is tjerefore shown on Fig. 3c (page 7).

From three examples, settling time is shortest in Example 3 where time required for the deadbeat response is $2\pi/3\omega$.

In the latter two examples, overdamped compensation circuit is introduced wheh the second (last) delayed step is connected in to the system. Fig. 2c (page 10) shows a scheme for introducing tachometer feedback with two delayed step input.

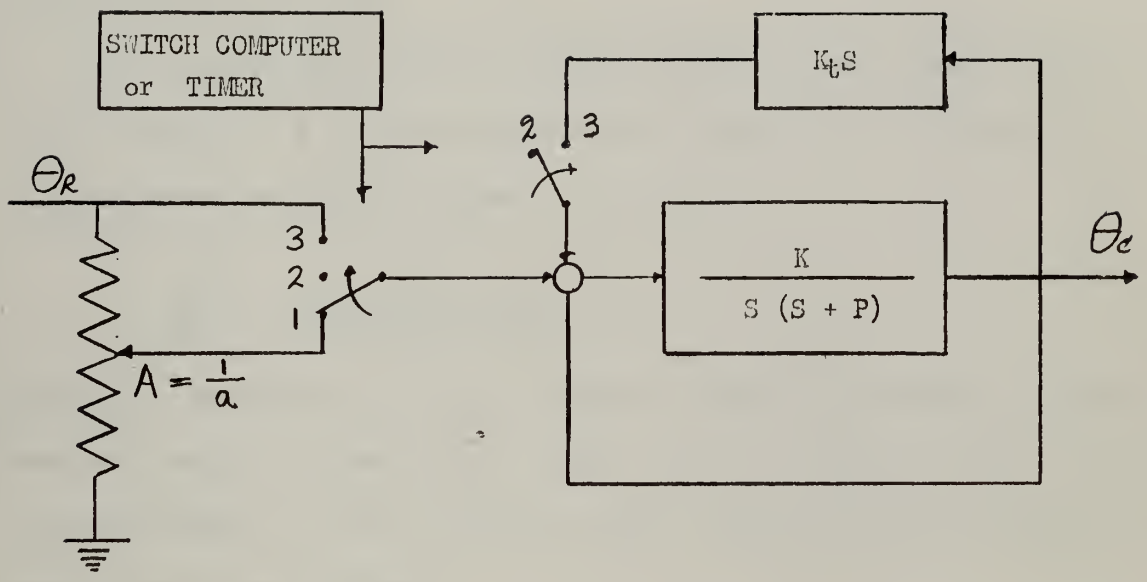
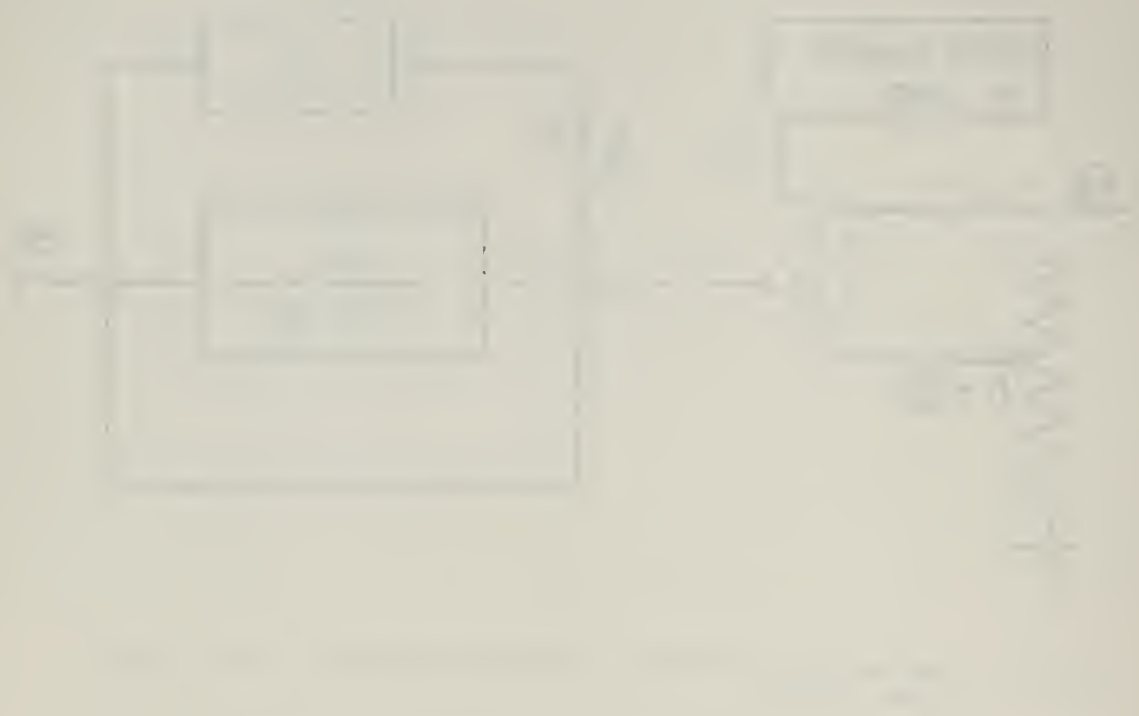


Fig. 2. Cascade and Feedback Compensation Schemes for a Second Order System.

c) Two Delayed Steps with Feedback Compensator.



SECTION 2 THIRD ORDER SYSTEM

2.1. THEORY FOR THIRD ORDER SYSTEMS

This discussion is applied only to the case in which the transfer function poles are real.

When complex poles exist the basic theory is valid theory is valid but the design of a cascade compensator is necessarily different. Consider a system with open loop transfer function.

$$\frac{\theta_c}{E} = \frac{K}{S(S+\alpha)(S+\beta)} \tag{15}$$

where K is large and the system is poorly damped or unstable. To apply posicast control the peak time and peak overshoot may be computed and the step subdivided so that the system reaches correspondence at exactly the peak of the first overshoot.

Assuming that K is set to place the complex roots on the imaginary axis, the characteristic equation is

$$S^3+(\alpha+\beta)S^2+\alpha\beta S+K=0 \tag{16}$$

Applying Routh's criterion, for pure imaginary roots

$$K=\alpha\beta(\alpha+\beta) \tag{17}$$

and the three roots are at $-(\alpha+\beta)$ and $\pm j\sqrt{\alpha\beta}$. The system function is

$$\frac{\theta_c}{\theta_R}(S) = \frac{K}{(S\pm j\sqrt{\alpha\beta})(S+\alpha+\beta)} \tag{18}$$

and transient response to a step input. Au(t) is

$$\theta_c(t) = A - A\frac{\gamma}{M}\cos(\omega t-\psi) - A\frac{\omega^2}{M^2}e^{-\gamma t} \tag{19}$$

where

$$\psi = \tan^{-1}\omega/\gamma$$

$$\omega = (\alpha\beta)^{\frac{1}{2}}$$

$$\gamma = \alpha + \beta$$

$$M = [(\alpha+\beta)^2 + \alpha\beta]^{\frac{1}{2}} = (\gamma^2 + \omega^2)^{\frac{1}{2}}$$

Fig. 4 (page 13) shows a root locus of the system function. Solving the differential equation, it can be shown that the peak time is approximately

$$T_p = (\pi + \psi)/\omega \tag{20}$$

The first maximum overshoot is

$$M_{pt} = \left|\frac{\theta_c}{\theta_R}(t)\right|_{max.} = A\left(\frac{M+\gamma}{M}\right) \tag{21}$$

And the initial amplitude required of the step for posicast control is

$$A\,u(t) = (M/M+\gamma)\,u(t) \tag{22}$$

Using parameters defined following Equation 19, $\gamma T_p > 2\pi$ and $\frac{\omega^2}{M^2} < \frac{1}{3}$. Therefore third term in Equation 19 is neglected at the neighborhood of $t = T_p$. If the error is reduced to zero at $t = T_p$ by making $\theta_R = 1.0$, the output does not remain stationary because the stored energy due to the third pole does not permit a discontinuous change in acceleration.

Thus there exists a boundary value (or initial condition) problem which is inherent in the posicast technique. Some solution is required to obtain a deadbeat response, even if compensation to an overdamped condition is not attempted. Three techniques are employed in this thesis:

a) The stored energy is discharged directly.

b) A pulse of finite amplitude and duration is injected. This produces a transient which cancels the transient due to the boundary condition.

c) The boundary condition problem and the compensation are combined,

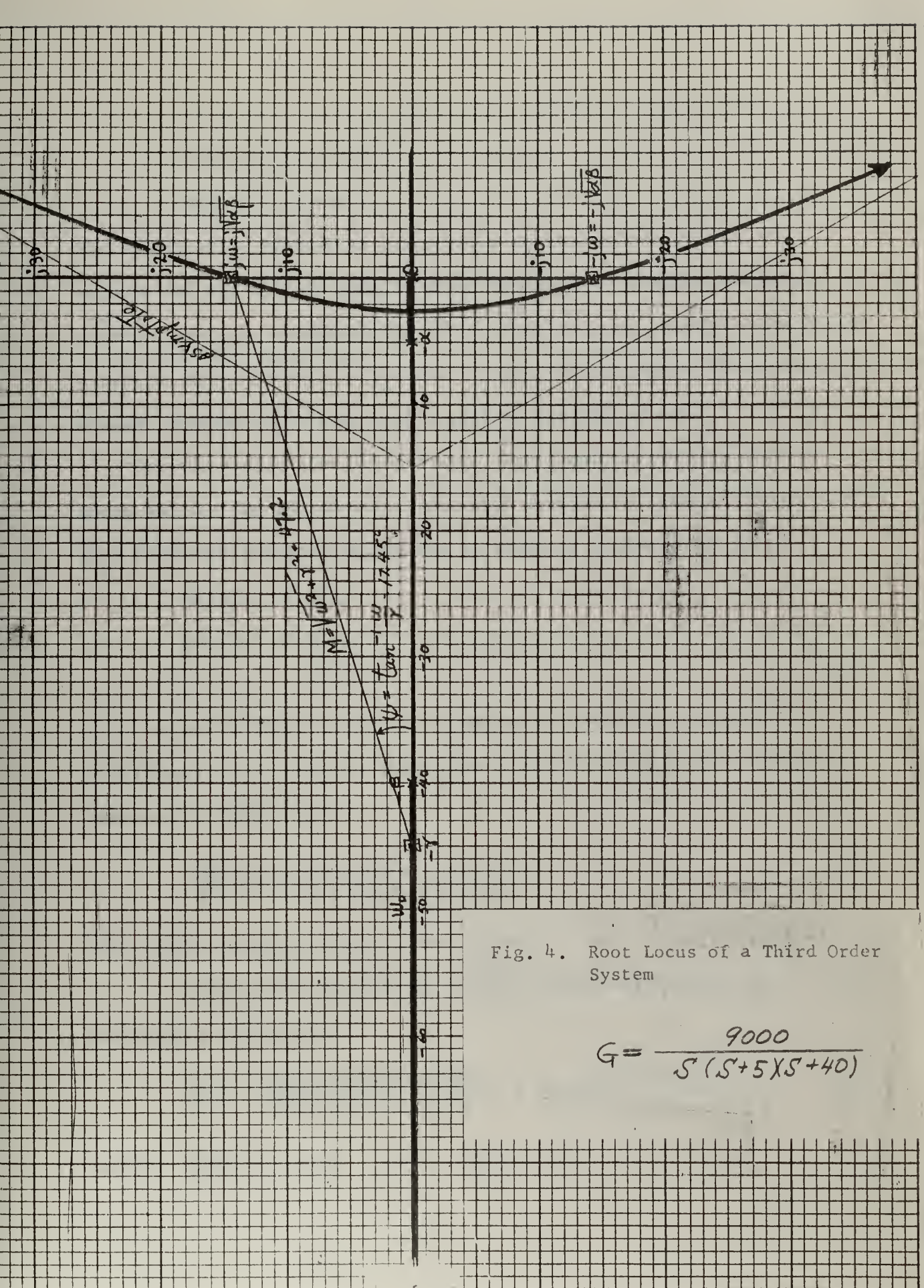


Fig. 4. Root Locus of a Third Order System

$$G = \frac{9000}{S(S+5)(S+40)}$$

and a phase space study of the compensated system shows that an eigenvector on the $\ddot{E}$ vs. $\dot{E}$ plane may be used to obtain the desired deadbeat response.

The compensation problem is essentially an independent problem when methods (a) and (b) are used to solve the boundary condition problem. Either cascade or feedback compensation may be used, though feedback compensation seems preferable because it does not compromise the static gain. Further discussion follows with regard to specific cases.

For the analysis of the problem, time domain is divided into two parts and for the second step input with overdamped compensation at time (T_p) there are boundary values due to posicast control. It is

$$\begin{aligned} \theta_c(T_p) &= \theta_{ci} \\ \dot{\theta}_c(T_p) &= \dot{\theta}_{ci} \\ \ddot{\theta}(T_p) &= \ddot{\theta}_{ci} \end{aligned} \tag{23}$$

Using cascade compensation, the compensator transfer function is $(s+\alpha)/(s+\omega_b)$ and a suitable equivalent transfer function for overdamped operation is known to be

$$\frac{\theta_c}{E}(s) = \frac{K}{s(s+\beta)(s+\omega_b)} \tag{24}$$

Then system function with boundary value is

$$\left[s^3+(\alpha+\beta)s^2+\alpha\beta s\right]\theta_c(s) = K\frac{s+\alpha}{s+\omega_b}E(s)+(s+\alpha)(s+\beta)\theta_{ci}+\ddot{\theta}_{ci} \tag{25}$$

Rearranging the equation

$$\theta_c(s) = \frac{K\theta_R(s)+(s+\beta)(s+\omega_b)\theta_{ci}}{s(s+\beta)(s+\omega_b)+K} + \frac{(s+\omega_b)\ddot{\theta}_{ci}}{(s+\alpha)\left[s(s+\beta)(s+\omega_b)+K\right]} \tag{26}$$

And if $\theta_R(t) = 1.0\,u(t)$ and $\theta_{ci} = 1.0$

$$\frac{K G_R(s) + (s+\beta)(s+\omega_D)\,\theta_{ci}}{s(s+\beta)(s+\omega_D) + K} = \frac{1}{s} \tag{27}$$

And Equation 25 reduces

$$\theta_c(s) = \frac{1}{s} + \frac{(s+\omega_D)\,\ddot{\theta}_{ci}}{(s+\alpha)[s(s+\beta)(s+\omega_D) + K]} \tag{28}$$

With the feedback compensation

$$\theta_c(s) = \frac{1}{s} + \frac{\ddot{\theta}_{ci}}{s(s+\beta)(s+\omega_D) + K} \tag{29}$$

In Equations 27 and 28, second term is the transient by the stored energy due to boundary value of acceleration.

2.2. COMPUTER STUDIES

Fig. 5a (page 17) shows a computer circuit with which posicast control is applied and stored energy causing an acceleration at $t = T_p$ is shorted out by a switching device.

Note that two relays are used, one marked SW1 to apply the posicast principle, and the other marked SW2, to cancel the boundary condition. Both relays operate from the output velocity signal which is readily obtained for most physical systems.

SW1 is operated at $t = T_p$, thus applying the posicast principle. SW2 is closed until $t = T_p\ (+)$. This shorts out the indicated capacitor, reducing to zero, then opens. The slight delay is obtained by experimental adjustment of the relay amplifier. In a physical system familiar switching arrangements may be used to short capacitors or open inductive storage elements.

The design of the cascade compensator is accomplished in the usual fashion. If overdamping is desired but the static gain may be reduced a single lead section with zero used to cancel the smaller hardware pole is usually satisfactory.

The attenuation of the filter is primarily responsible for the real roots obtained in this fashion. If overdamping is required but gain reduction is not permissible several cascaded sections may be necessary. All of the usual compensation design techniques apply, and it should be noted that there is no stored energy in the compensator when it is inserted in the system, and zero voltage is applied to the compensator at the instant of insertion.

Calculation for the computer set up. Block diagram of underdamped system is shown in Fig. 6(a) (page 18) the parameters are K = 9000,

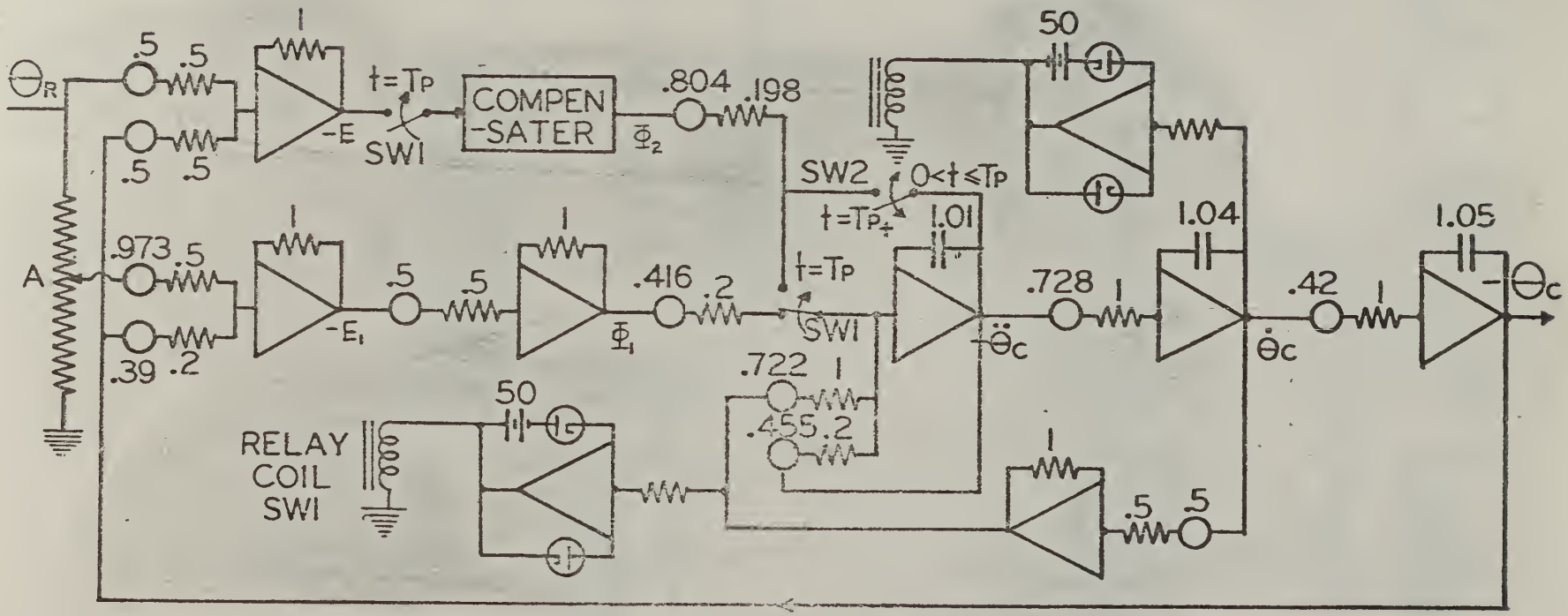


Fig. 5. Circuit for a Third Order Servo. $G = \frac{K}{S(S+\alpha)(S+\beta)}$

a) Series Compensation, Acceleration Boundary Condition Shorted Out.

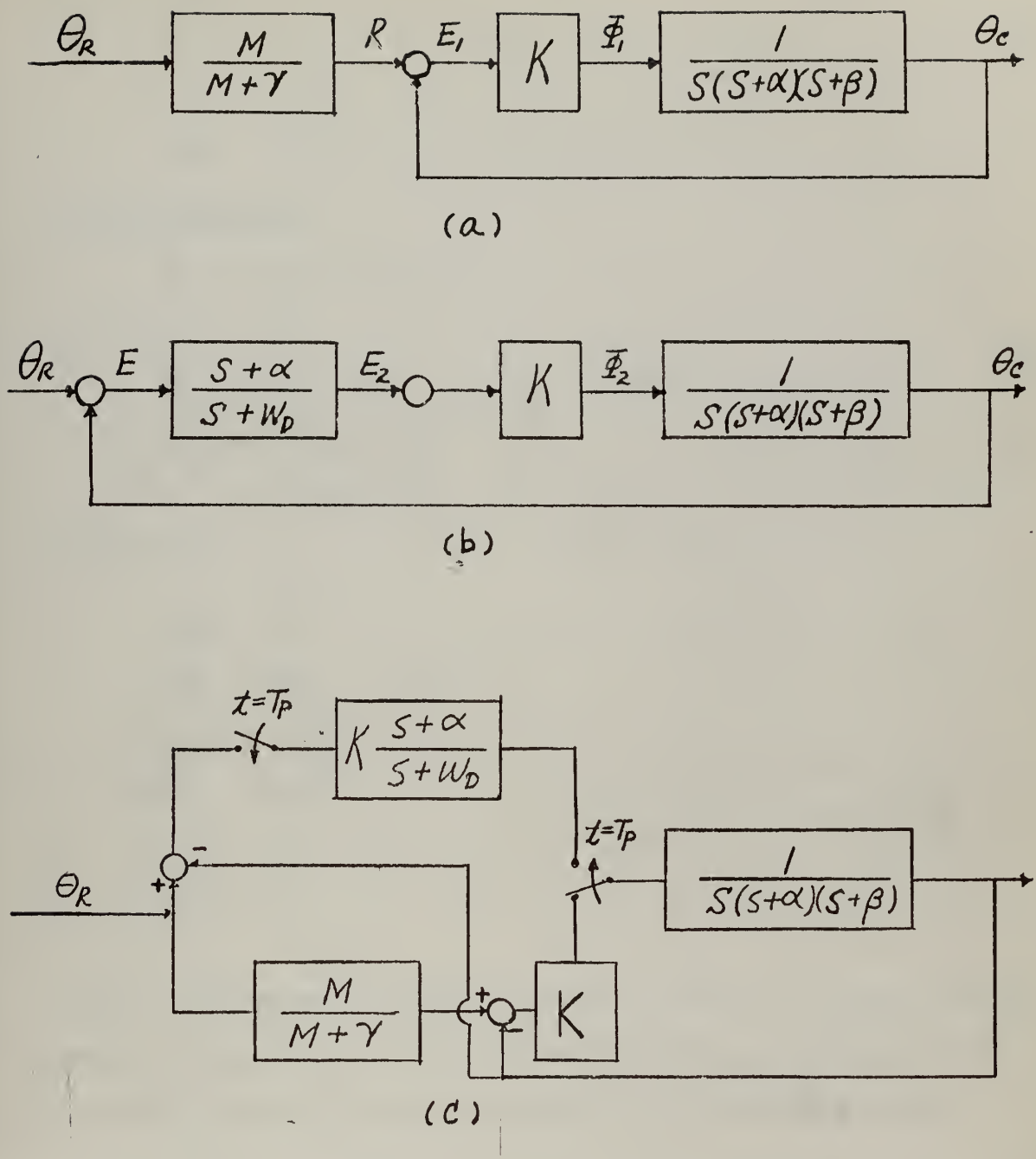


Fig. 6. Block Diagrams for a Third Order System.

$\alpha = 5$, $\beta = 40$. From Equations 19 and 20

$$\theta_c(t) = .512 - .477 \cos(5.11t - 1.41) \quad 04\ell^{-}$$
$$\dot{\theta}_c(t) = 6.7 \sin(5.11t - 1.41) - 2.\ell^{-4.4t} \tag{30}$$
$$\ddot{\theta}_c(t) = 97.6 \cos(5.11t - 1.175) + 44.\ell^{-4.4t}$$

Underdamped System

$$E_1 = (M/M+1)\theta_R - \theta_c$$
$$T_1 = KE_1$$
$$\ddot{\theta}_c = (1/S)[T_1 - (\alpha+\beta)\dot{\theta}_c - \alpha\beta\theta_c] \tag{31}$$
$$\dot{\theta}_c = (1/S)\ddot{\theta}_c$$
$$\theta = (1/S)\dot{\theta}_c$$

Scalings are

$$\alpha_R = 20$$
$$\alpha_{\theta R} = \alpha_{\theta c} = .0125$$
$$\alpha_{E_1} = .0064$$
$$\alpha_{T_1} = 57.6$$
$$\alpha_{\ddot{\theta}_c} = 1.4$$
$$\alpha_{\dot{\theta}_c} = .1$$

At the first maximum overshoot, full step input and overdamped compensation is connected to the system, and resulting system is shown in Fig. 6(b) (page 18) with block diagram. The system function is

$$\frac{\theta_c}{\theta_R}(S) = \frac{K}{S(S+\beta)(S+\omega_n) + K} = \frac{900}{(S+.045)(S+23.33)(S+33.7)} \tag{32}$$

And transient response to a step input, u(t) is

$$\theta_c(t) = 1 - 1.046\ell^{-.045t} + .55\ell^{-23.33t} - .09\ell^{-33.7t} \tag{33}$$

Overdamped System

$$E = \theta_R - \theta_c$$

$$E_2 = \frac{S+\alpha}{S+\omega_D} E = E - \frac{\omega_D - \alpha}{S+\omega_D} E \qquad (34)$$

$$\Phi_2 = K E_2$$

Scalings are $\alpha_E = \alpha_{E_2} = .0125 \qquad \alpha_{\Phi_2} = 112.5$

Fig. 5 (b) (page 21) shows a computer circuit with which posicast control is applied and stored energy is shorted out by switching device. Arranging the transfer function somewhat similar to the physical system. Then the relations are for $0 \leq t \leq T_p$ (underdamped system)

$$E_1 = (M/M+r)\,\theta_R - \theta_c$$

$$\Phi_1 = K E_1$$

$$T = \left(\frac{1}{S+\beta}\right) \Phi_1 \qquad (35)$$

$$\dot{\theta}_c = \left(\frac{1}{S+\alpha}\right) T$$

$$\theta_c = (1/S)\,\dot{\theta}_c$$

Where

$$T(S) = (S+\alpha) S\, \theta_c(S) \qquad (36)$$

Then

$$T(t) = 52.7 \sin(81^\circ t + 53^\circ) + 42.4\, e^{-45t} \qquad (37)$$

Scaling is $\alpha_T = 1.4$ and set up for $t > T_p$ is same as before.

Fig. 5 (c) (page 22) shows a computer circuit which simulates the energy storage elements in transfer function form, and which effectively cancels the stored energy by injecting a pulse at the switching instant. Computer set up is same as in Fig. 5 (b) except the branch of generating a pulse.

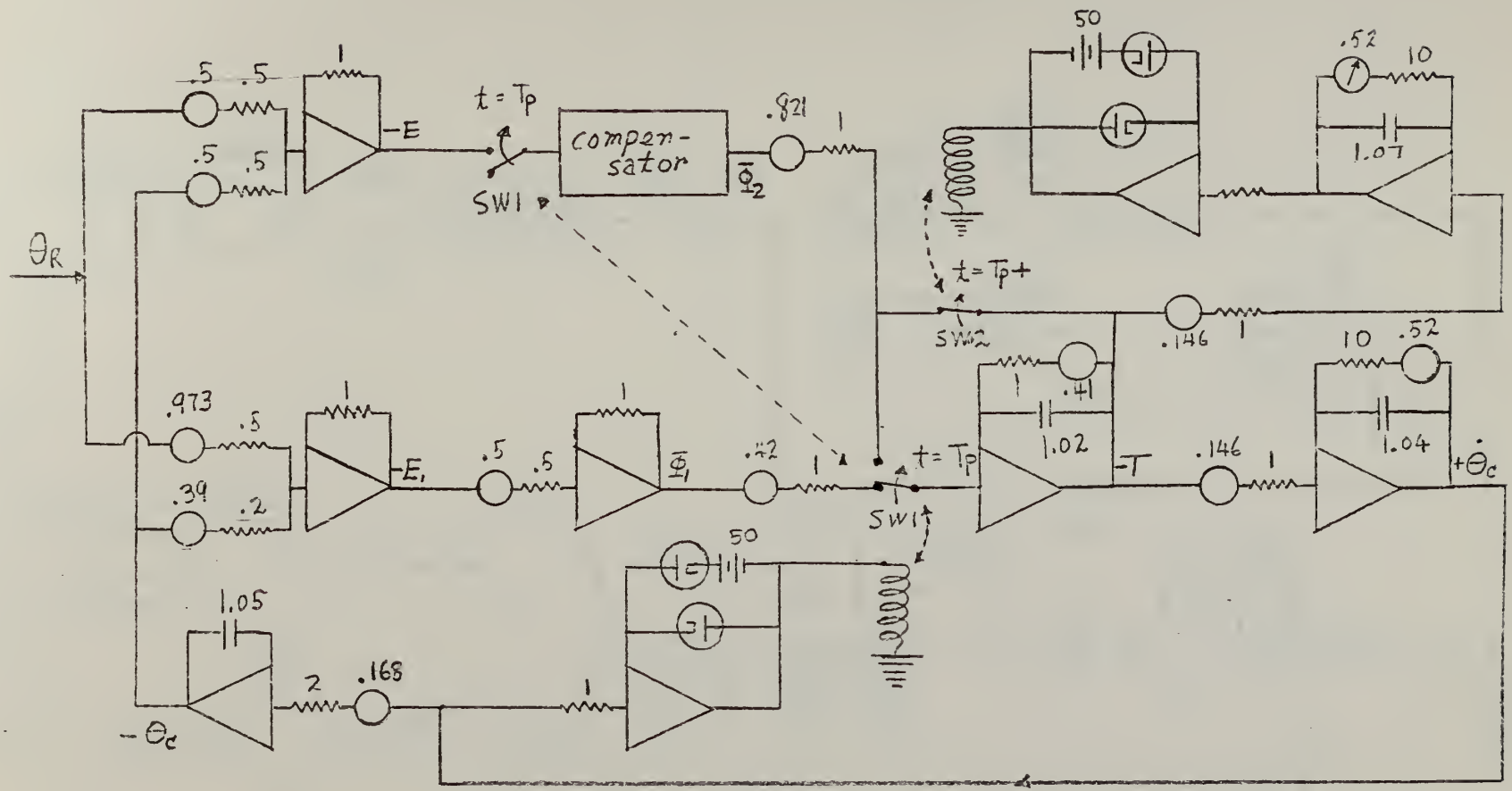


Fig. 5. Circuit of a Third Order Servo. $G = \frac{K}{S(S+\alpha)(S+\beta)}$

b) Series Compensation, Boundary Condition Shorted Out.

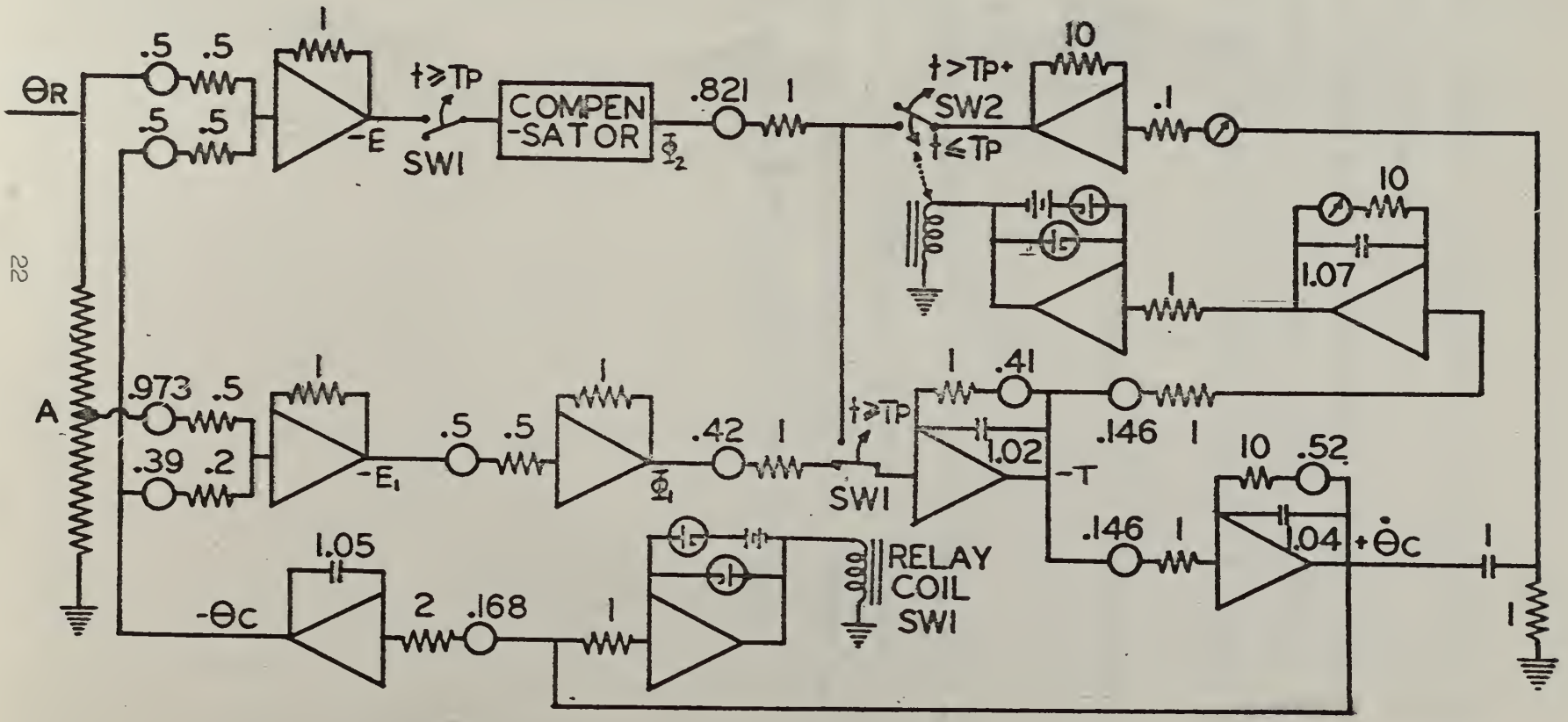


Fig. 5. Circuit for a Third Order Servo. $G = \frac{K}{S(S+\alpha)(S+\beta)}$

c) Series Compensation, Stored Energy Cancelled by Applied Pulse.

All of the systems of Fig. 5 were tested with step inputs. The parameters were the same.

The results obtained were identical, and a single set of results is presented on Fig. 7 (page 24).

To design feedback compensation, assuming that the transfer function of the underdamped system is given by Equation 15, and that a suitable equivalent transfer function for overdamped operation is known to be Equation 24.

It is readily shown that a suitable feedback function converting equation 16 to equation 24 is

$$H = (1/K)\left[(\omega_b - \alpha)\, S^2 + \beta(\omega_b - \alpha)\, S\right] \qquad (38)$$

To obtain overdamped operation it is necessary to connect this feedback path when the second step of the posicast control is applied. It may be noted that the static gain is not altered when the feedback compensator is introduced, but the boundary condition problem remains.

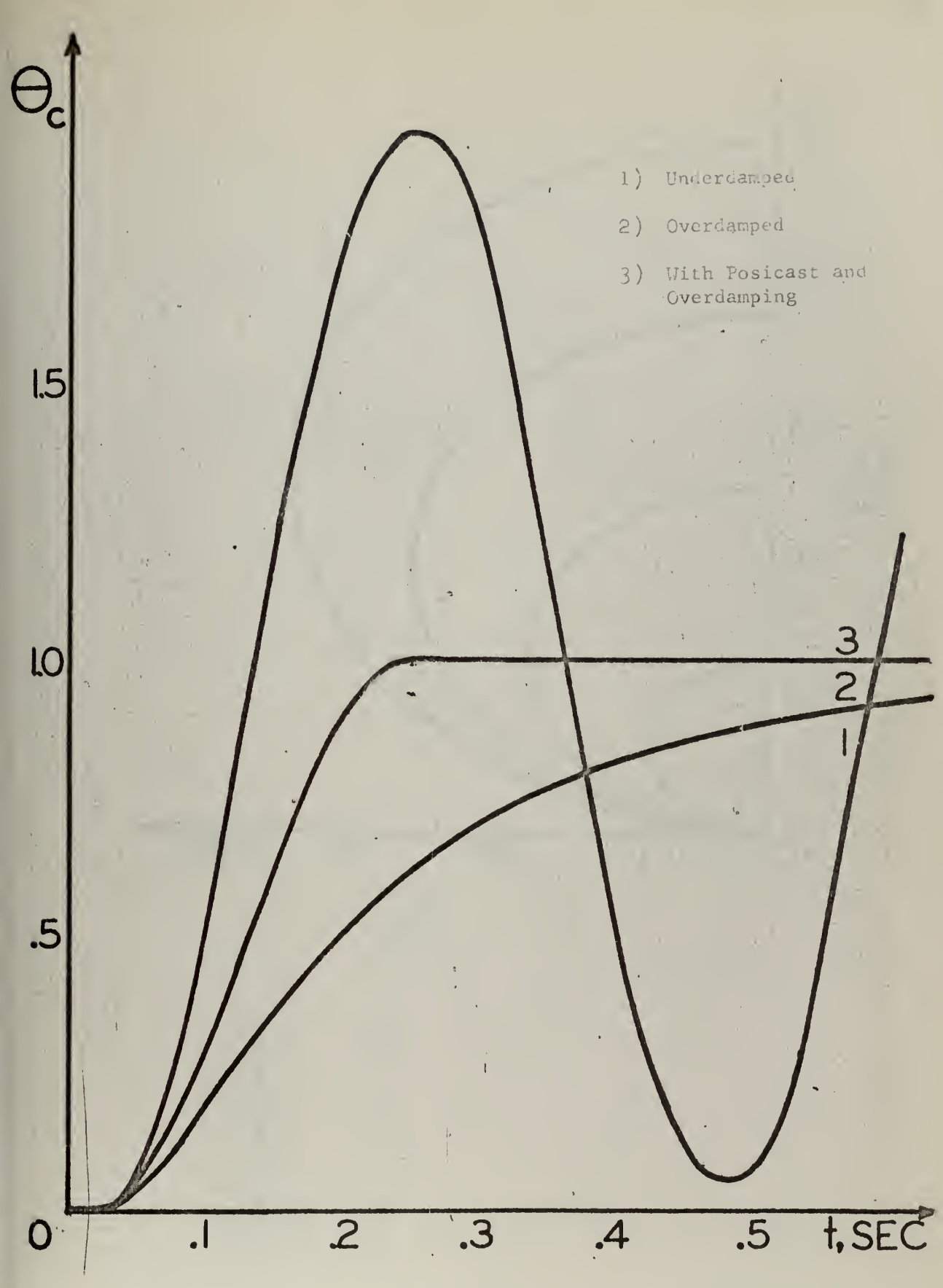
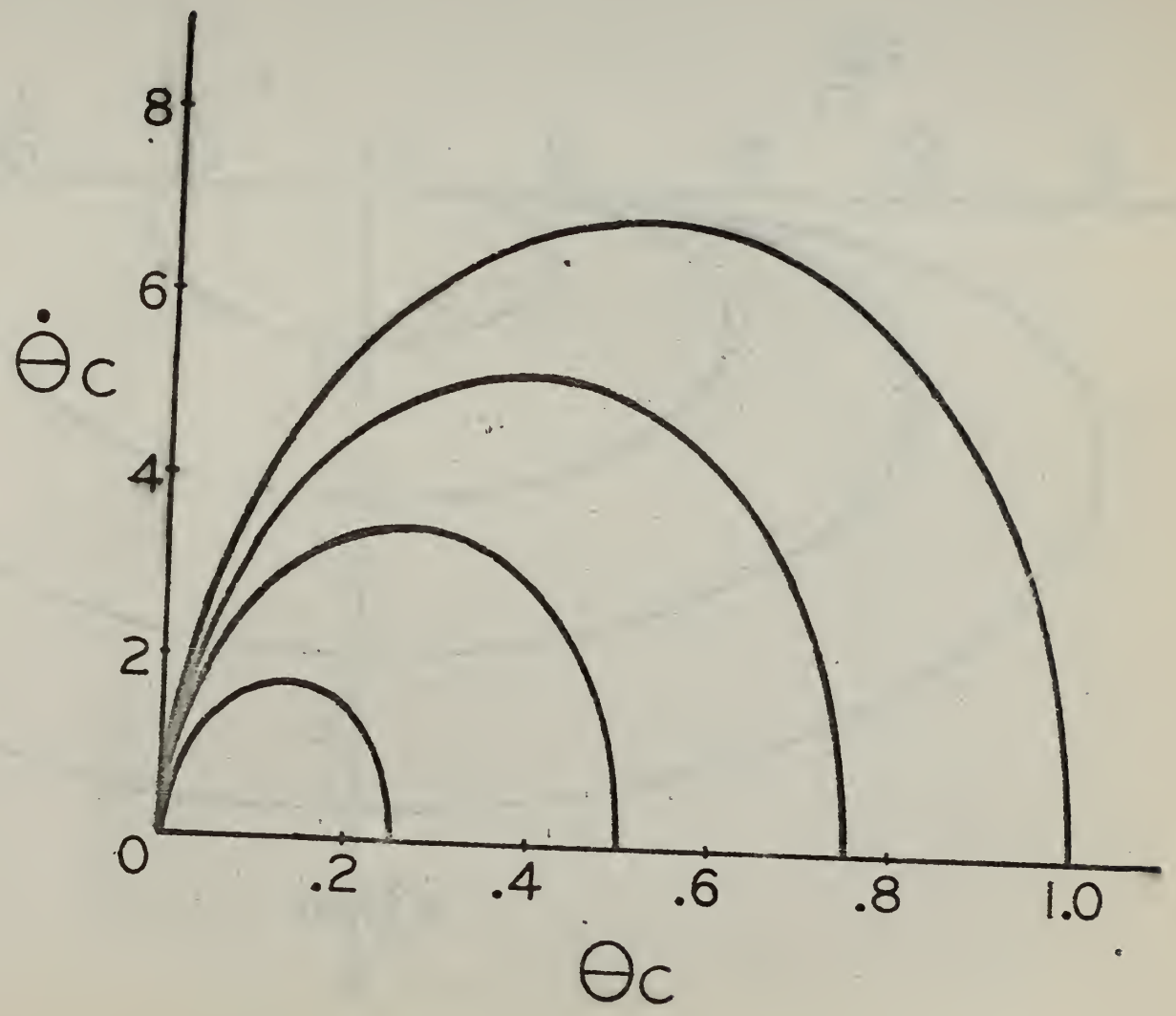


Fig. 7. Posicast Control of a Third Order System Using Techniques of Fig. 6.

a) Transient Response Curves.

Fig. 7. Posicast Control of a Third Order System Using Techniques of Fig. 6.

b) $\dot{\theta}_c$ vs. θ_c Phase Plane.



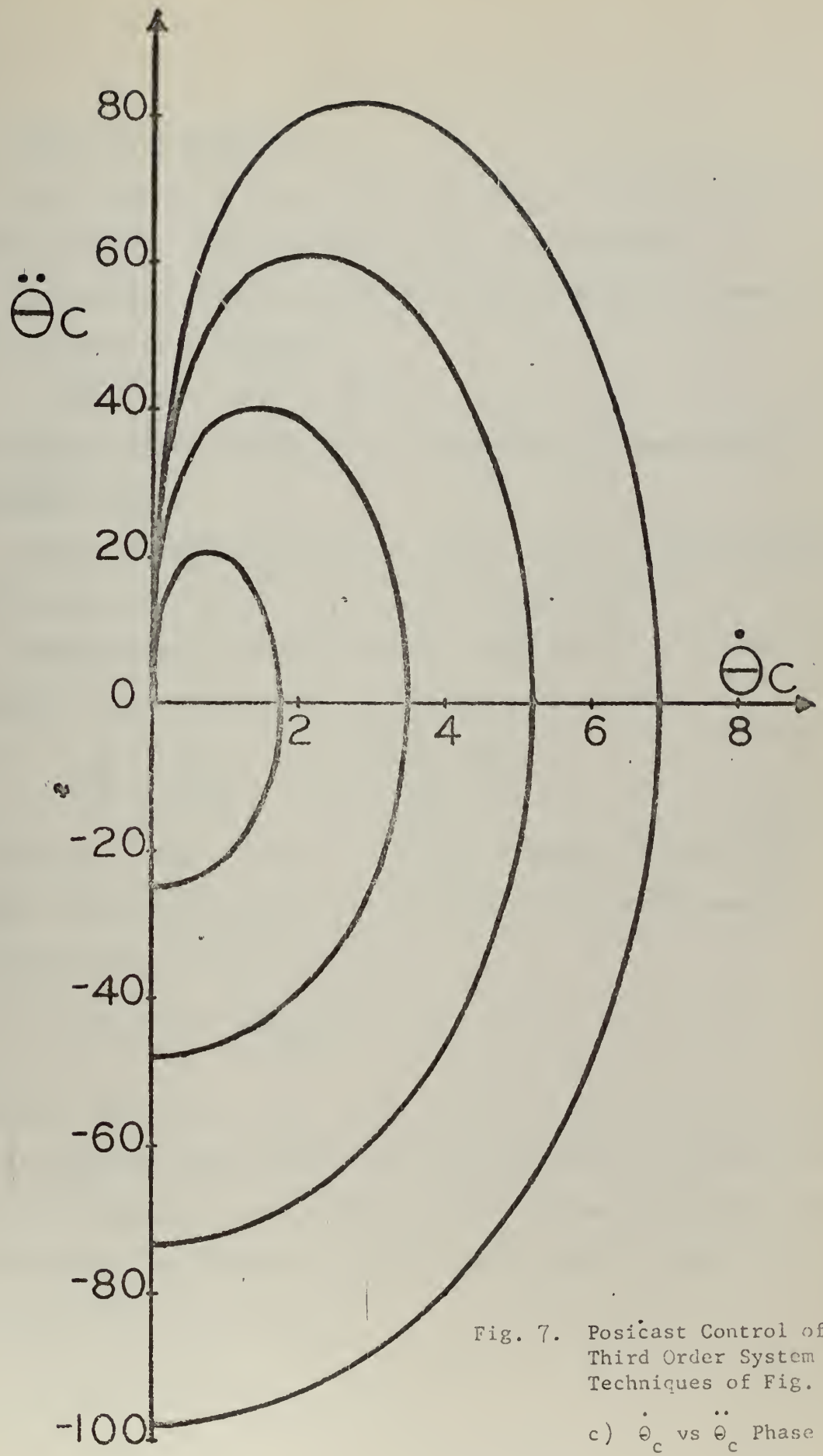


Fig. 7. Posicast Control of a Third Order System Using Techniques of Fig. 6.

c) $\dot{\theta}_c$ vs $\ddot{\theta}_c$ Phase Plane.

2.3. PHASE SPACE TECHNIQUE

While the methods of Fig. 5 are applicable a new and interesting technique evolves from a phase space study of the problem.

For the third order system the differential equation in terms of the error variable is of the form

$$\dddot{E} + X\ddot{E} + Y\dot{E} + KE = 0 \tag{39}$$

The numerical values of X and Y are different for the underdamped and overdamped cases.

For the underdamped system $X = \alpha + \beta$, $Y = \alpha\beta$, $K = \alpha\beta(\alpha + \beta)$ and for overdamped system $X = \beta + \omega_D$, $K = \alpha\beta(\alpha + \beta)$.

The phase space defined by Equation 39 is the E, $\dot{E}$, $\ddot{E}$, space. A component of the slope of the phase trajectory is given by

$$\frac{\dddot{E}}{\ddot{E}} = \frac{d\ddot{E}}{d\dot{E}} = -\frac{X\ddot{E} + Y\dot{E} + KE}{\ddot{E}} \tag{40}$$

by choosing a constant value $N = d\ddot{E}/d\dot{E}$, a phase is defined in the phase space. By further imposing the condition $E \equiv 0$ the phase space is collapsed to the $\ddot{E}$ vs. $\dot{E}$ plane and

$$N = -\frac{X\ddot{E} + Y\dot{E}}{\ddot{E}} \tag{41}$$

defines a line in the $\ddot{E}$ vs. $\dot{E}$ plane which is a straight line through the origin with the property that at any point on this line the slope of the $\ddot{E}$ vs. $\dot{E}$ trajectory has the designated value N. When the second order system defined by Equation 41 has real roots, then two eigenvectors are defined in the $\ddot{E}$ vs. $\dot{E}$ plane.

Manipulating Equation 41

$$\frac{\ddot{E}}{\dot{E}} = -\frac{Y}{N+X} \triangleq N \tag{42}$$

thus the values of N which define the eigenvectors are

$$N = \frac{-X \mp (X^2 - 4Y)^{\frac{1}{2}}}{2} \tag{43}$$

For the underdamped system this evaluates to N = $-\alpha$, $-\beta$, and for the overdamped system N = $-\beta$, $-\omega_D$.

The physical interpretation of these results is simple. If the underdamped system is set into motion by any forcing function (such as a step), and at an instant when $\ddot{E}/\dot{E} = N = -\alpha(or\ \beta)$ the error signal E is forced to zero, such as by opening the error channel, then the system will drift to standstill with $\ddot{E}$ and $\dot{E}$ going to zero simultaneously and at every instant the state point is on the straight line $\ddot{E}/\dot{E} = N$. In like manner, if the error signal is forced to zero and the feedback circuit connected simultaneously when $\ddot{E}/\dot{E} = N = -B$ (or $-\omega_D$) the overdamped system behaves in like manner. Fig. 8 (page 29) shows acceleration vs. velocity trajectories obtained in this fashion with the feedback compensated system shown in Fig. 9 (page 30).

To combine this concept with posicast control, the following procedure applies: the initial step magnitude is reduced to a value slightly less than that which would be used with posicast control; the damping circuits are connected and the error signal forced to zero when the ratio $\ddot{E}/\dot{E}$ is at the predetermined value; when the system stops the error channel is reconnected and the full input signal (second step) is used, the system is in correspondence with the feedback

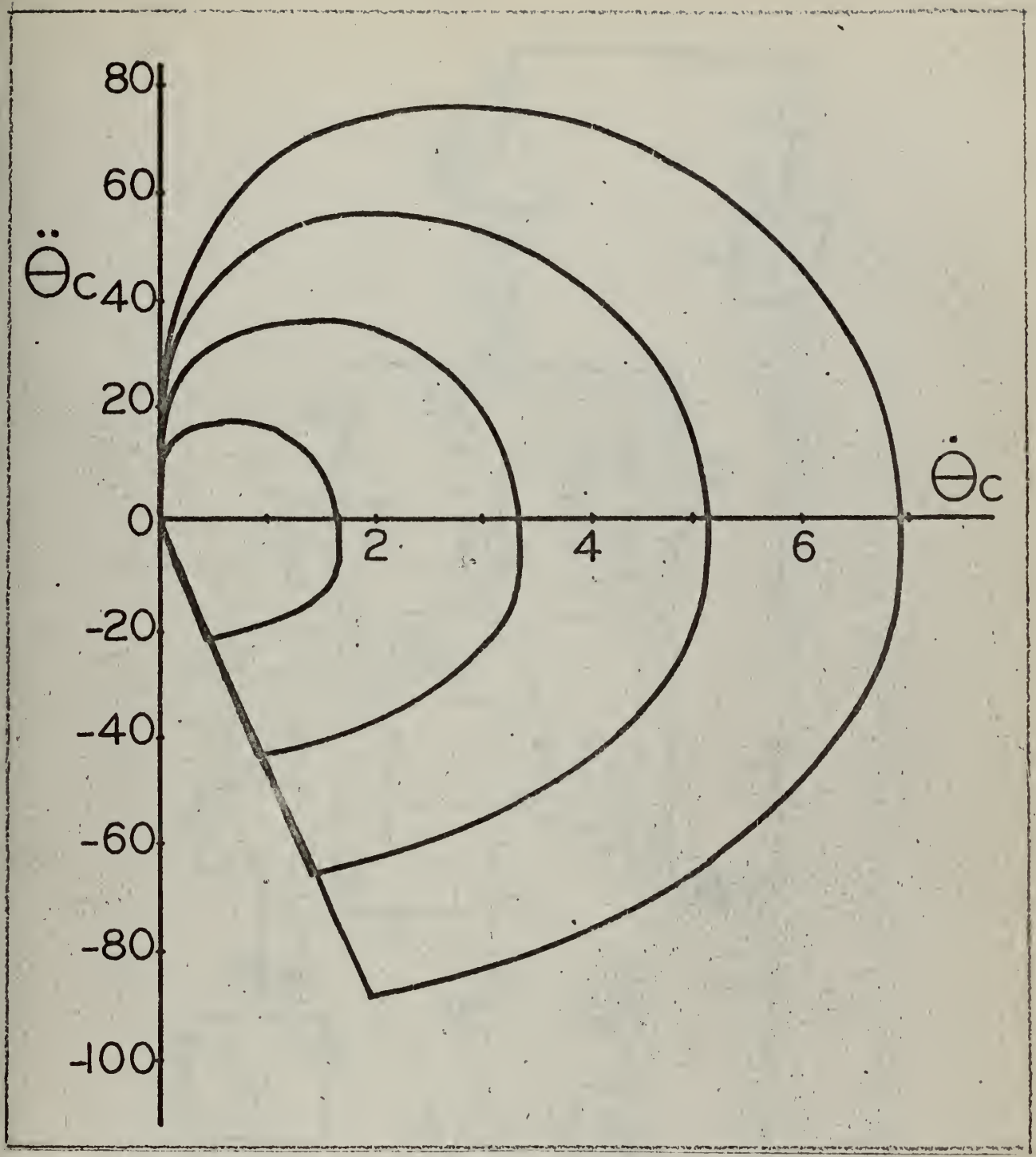


Fig. 8. Acceleration vs Velocity Trajectories with Damping Connected and Error Signal Cancelled at $N = -\omega_D$.

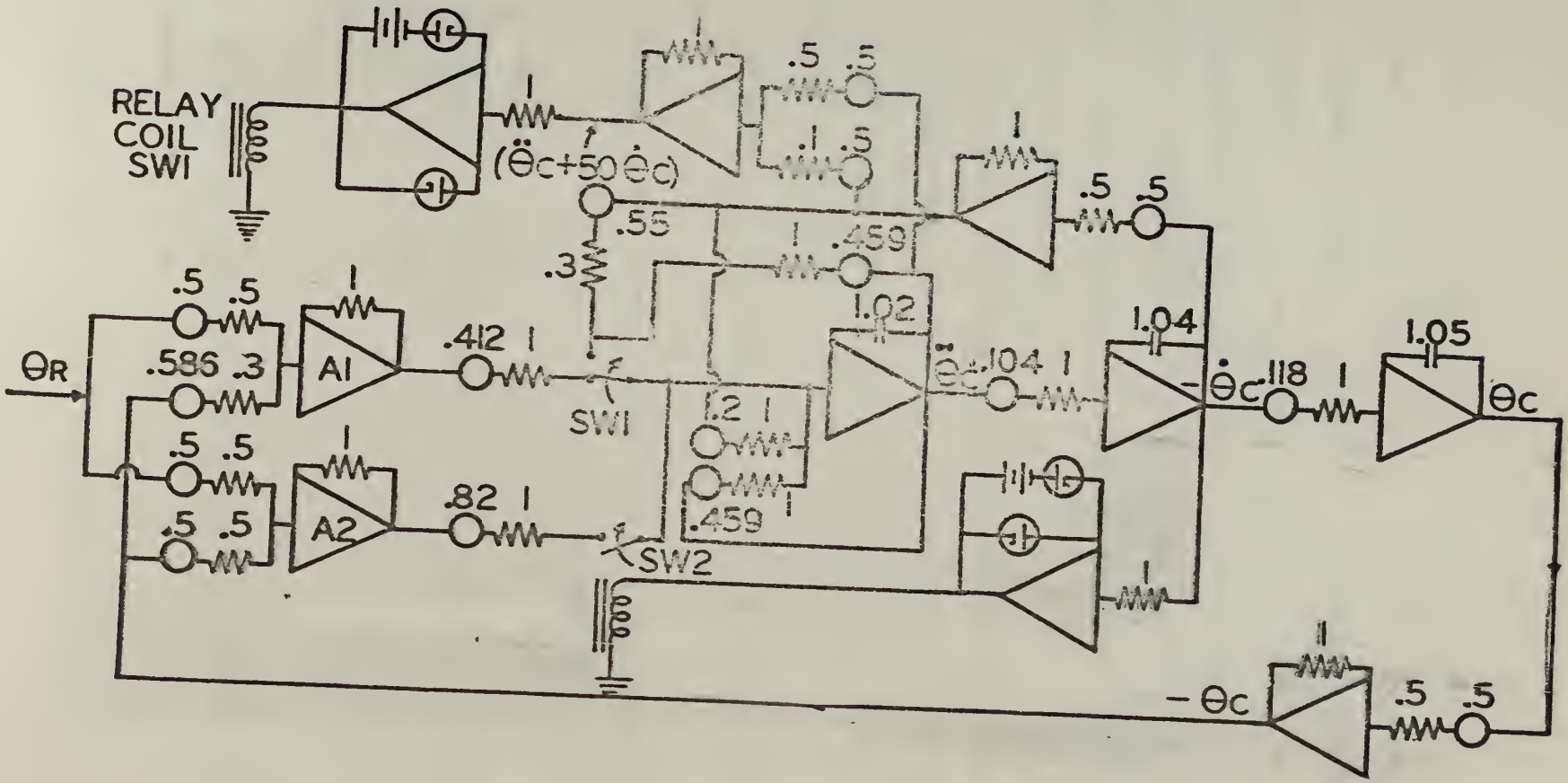


Fig. 9. Computer Diagram for Third Order System Using the Phase Space Technique.

damping circuit is connected as desired. The reduction in the amplitude of the initial step is due to the fact that the dynamics of the system are changed when the theory of the $\ddot{E}$ vs. $\dot{E}$ plane is applied. To obtain deadbeat response it is necessary that E, $\dot{E}$ and $\ddot{E}$ reaches zero simultaneously. By following the eigenvector in the $\dot{E}$ vs. $\dot{E}$ plane it is guaranteed that $\ddot{E}$ and $\dot{E}$ reaches zero simultaneously and quickly, but reduction of the error signal to zero in this manipulation reduced the system to open loop operation for a short time during which the physical error does not seek a null.

If the initial step required by posicast theory is applied, switching to the eigenvector in the $\ddot{E}$ vs. $\dot{E}$ plane permits a slight overshoot in position due to the change in the velocity function. Therefore a smaller initial step is required.

Computations for adjustment of posicast step magnitude are as follows:

Consider an underdamped system described by Equation 15 with $\alpha = 5$, $\beta = 40$, K = 9000, and overdamped system described by Equation 24 with $\beta = 40$, $\omega_b = 50$ and K = 9000.

The chosen eigenvector on the $\ddot{E}$ vs. $\dot{E}$ plane for damping purpose is

$$\ddot{E}/\dot{E} = N = -\omega_b = -50 \tag{44}$$

from which

$$\dot{E} + 50E = 0 \tag{45}$$

The solution of the differential equation for the output motion of the underdamped system gives:

$$\theta_c(t) = \frac{M}{M+J} - \frac{\gamma}{M+J}\cos(\omega t - \varphi) - \frac{\omega^2 e^{-\gamma t}}{M(M+J)} \tag{46}$$

$$\ddot{\theta}_c(t) = \frac{\omega^2\gamma}{M+\gamma}\sin(\omega t-\psi) + \frac{\gamma\omega^2 e^{-\gamma t}}{M(M+\gamma)} \tag{47}$$

$$\dot{\theta}_c(t) = \frac{\omega^2\gamma}{M+\gamma}\cos(\omega t-\psi) - \frac{\omega^2\gamma^2 e^{-\gamma t}}{M(M+\gamma)} \tag{48}$$

Where the roots of the equation are γ; $0 \pm j\omega$, and the other parameters are defined following Equation 19. At the instant of switching $t \cong (\pi+\psi)/\omega$ and the exponential terms may be neglected.

$$\ddot{\theta}_c + 50\dot{\theta}_c = 0 = \frac{\omega^2\gamma}{M+\gamma}\cos(\omega t-\psi) + \frac{50\omega\gamma}{M+\gamma}\sin(\omega t-\psi) \tag{49}$$

which reduces to

$$\tan(\omega t-\psi) = -\omega/50 = -.283 \tag{50}$$

from which a first approximation is $(\omega t-\psi) = 2.87_{rad} = 164.2^\circ$. Substituting this into the equation for $\theta_c(t)$

$$\theta_c = \frac{M}{M+\gamma} - \frac{\gamma}{M+\gamma}\cos 164.2^\circ = .981 \tag{51}$$

which is the value of the output displacement when the damping loop is closed and the error signal nullified. $\dot{\theta}_c$ and $\ddot{\theta}_c$ are readily computed in like manner so the initial conditions for the damping mode are $\theta_{ci} = .981$; $\dot{\theta}_{ci} = 1.88$ and $\ddot{\theta}_{ci} = -\omega_b\dot{\theta}_{ci} = -94.0$. Using these initial conditions in conjunction with Equation 24.

$$S(S+\beta)(S+\omega_b)\theta_c(S) = KE + (S+\beta)(S+\omega_b)\theta_{ci} + (S+\beta+\omega_b)\dot{\theta}_{ci} + \ddot{\theta}_{ci} \tag{52}$$

and noting that E is zero by definition,

$$\theta_c(S) = \frac{(S+\beta)(S+\omega_b)\theta_{ci} + (S+\beta+\omega_b)\dot{\theta}_{ci} + \ddot{\theta}_{ci}}{S(S+\beta)(S+\omega_b)} \tag{53}$$

then

$$\theta_c(t) = \theta_{ci} + \frac{\beta + \omega_b}{\beta\omega_b}\dot{\theta}_{ci} + \frac{\ddot{\theta}_{ci}}{\beta\omega_b} + \Sigma \tag{54}$$

where

$$\Sigma = -\left[\frac{\omega_b\dot{\theta}_{ci} + \ddot{\theta}_{ci}}{\beta(\omega_b - \beta)}\right]e^{-\beta t} + \left[\frac{\beta\dot{\theta}_{ci} + \ddot{\theta}_{ci}}{\omega_b(\omega_b - \beta)}\right]e^{-\omega_b t} \tag{55}$$

and these exponential terms are damped out quickly. Noting that in this damping mode $\ddot{\theta}_{ci} + \omega_b\dot{\theta}_{ci} = 0$. It follows that

$$\begin{aligned}\theta_c(t) &= \theta_{ci} + \frac{\beta\omega_b}{\beta\omega_b}\dot{\theta}_{ci} - \frac{\omega_b\ddot{\theta}_{ci}}{\beta\omega_b} \\ &= \theta_{ci} + \frac{\dot{\theta}_{ci}}{\omega_b}\end{aligned} \tag{56}$$

Inserting numbers

$$\theta_c(t) = .981 + \frac{1.88}{50} = 1.0186$$

This indicates about two percent overshoot. It is desired that

$\theta_c(t) = 1.0$ or

$$\theta_{ci} + \frac{\dot{\theta}_{ci}}{\omega_b} = 1.0$$

The magnitude of step used was $(M/M+\gamma) = .512$ Let the new magnitude be A, such that

$$\frac{A}{1.0} = \frac{.512}{1.0186}$$

from which A = .502 as a first approximation. The operation of Fig. 9 is now explainable. For a step displacement input θ_R is fed through A, which reduced the step magnitude, and through SW1. The underdamped system responds rapidly. When $\ddot{\theta} + 50\dot{\theta} = 0$ the switch SW1 operates,

opening the error channel and applying the feedback damping loops.

The open loop arrangement brings $\ddot{\theta}_e$ and $\dot{\theta}_e$ to zero simultaneously, and for proper adjustment of the posicast step $\theta_e = \theta_R$ when $\dot{\theta}_e = 0$. At this instant SW2 recloses the error channel and locks the system. Fig. 10a (page 35) shows a family of $\dot{\theta}$ vs. θ phase trajectories obtained with the system of Fig. 9. It is readily seen that they are essentially the same as those of Fig. 7b (page 25). Fig.10b (page 36) shows a family of step-response curves, indicating that the performance is satisfactory over a range of step sizes.

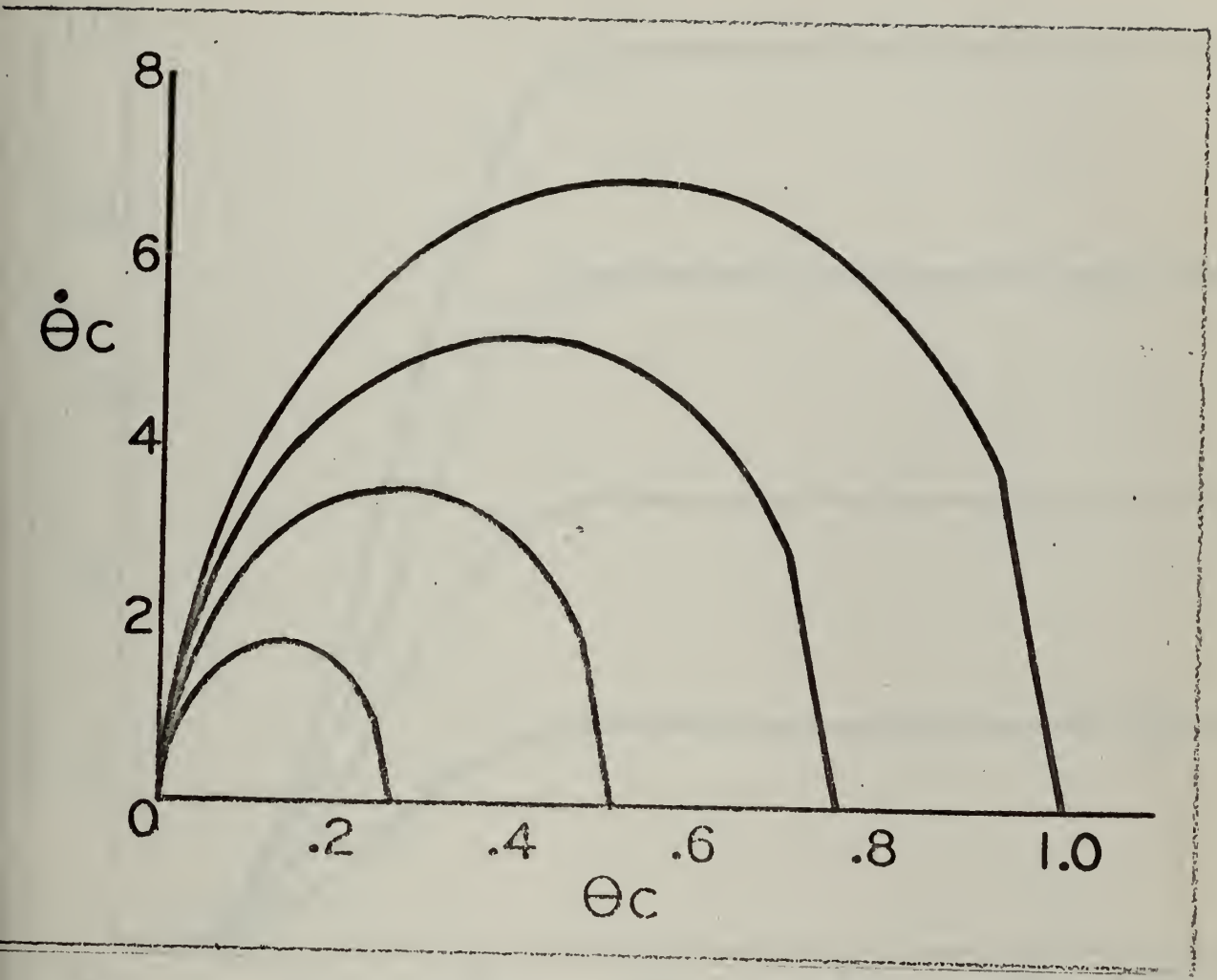


Fig. 10. Posicast Control of a Third Order System Using Phase Space Technique.

a) θ_c vs $\dot{\theta}_c$ Phase Plane Family.

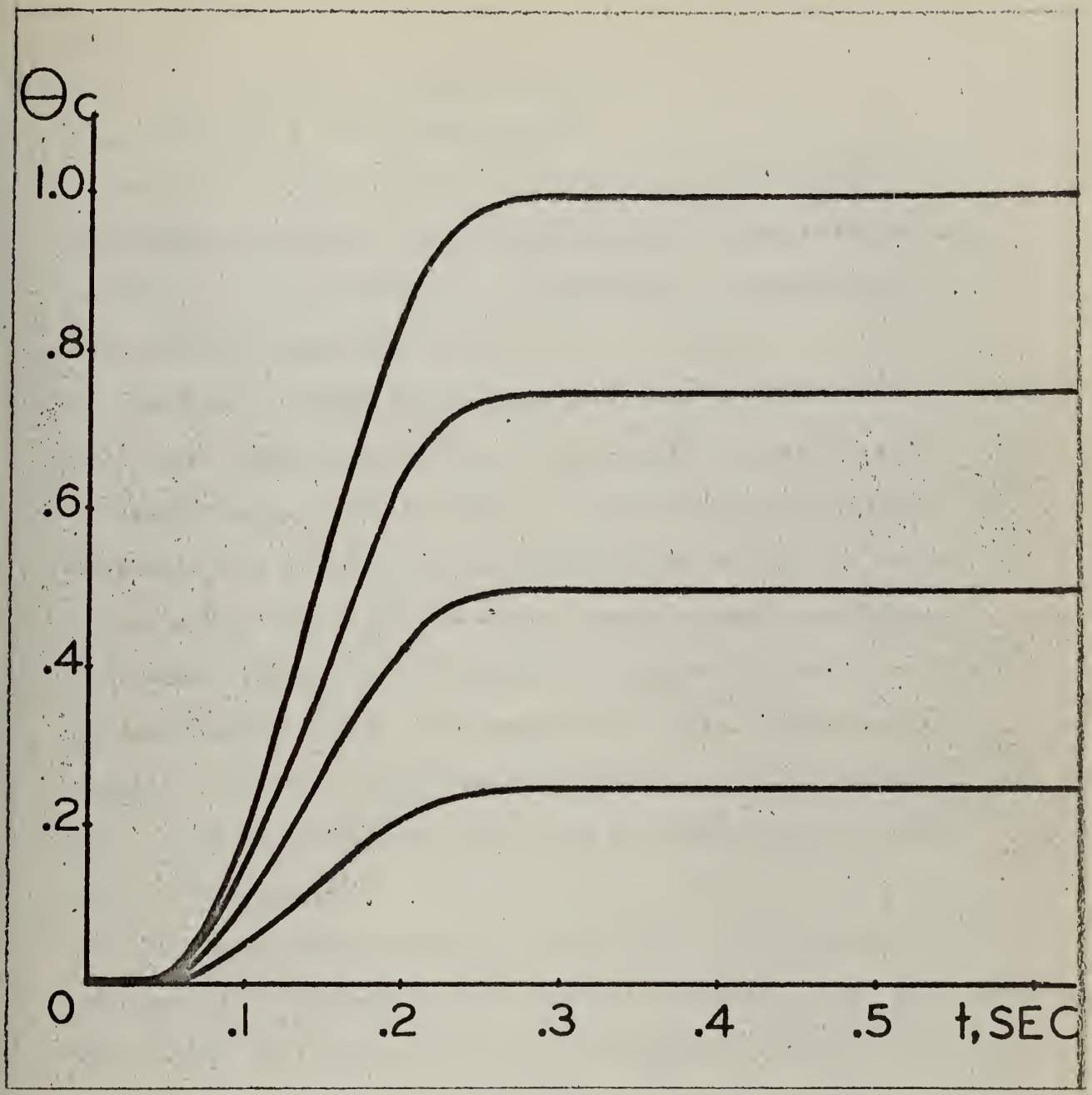
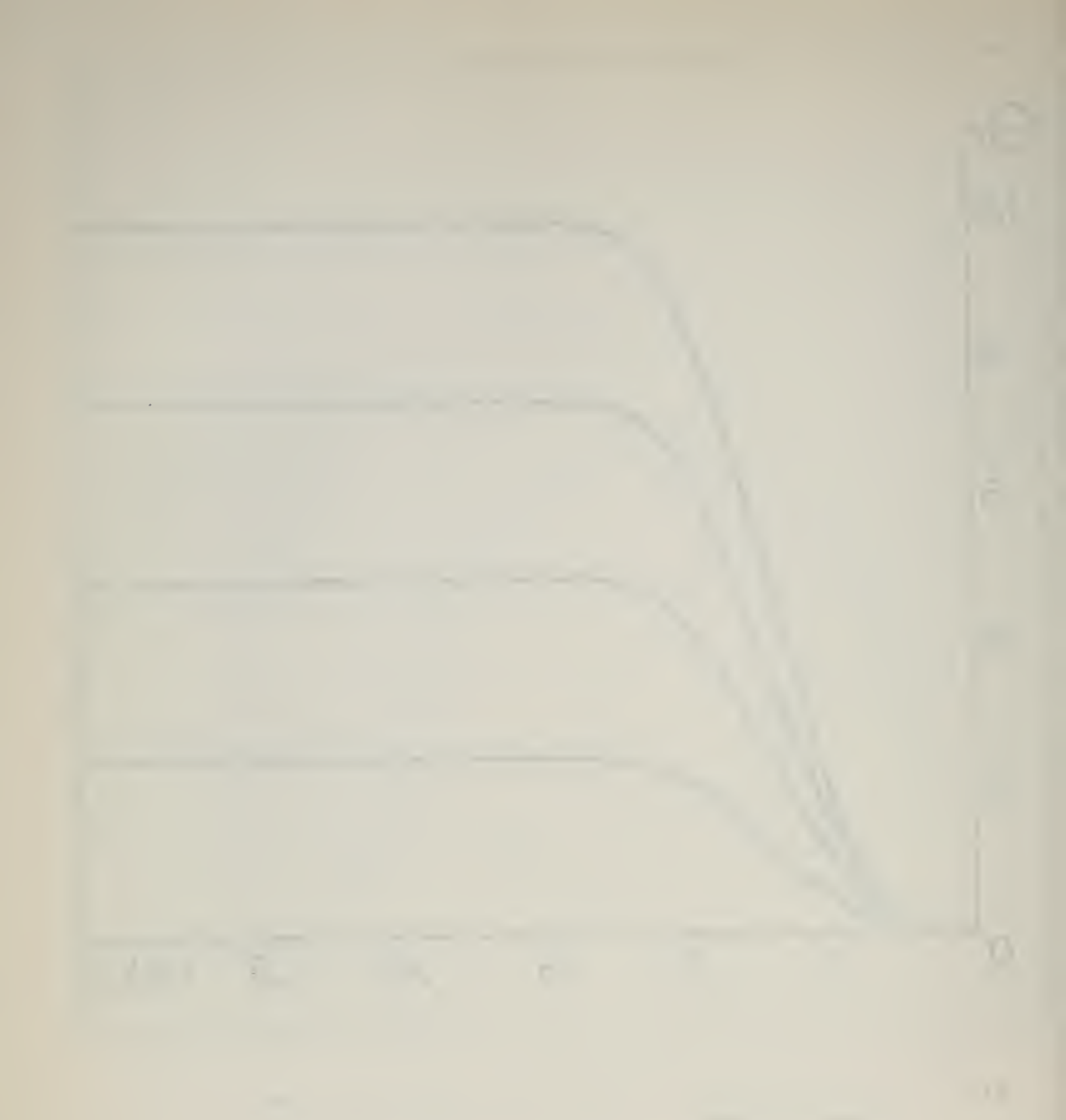


Fig. 10. Posicast Control of a Third Order System Using Phase Space Technique.

b) Step Response (Θ_c vs t) Family.



SECTION 3 FOURTH ORDER SYSTEM

3.1. THEORY FOR A FOURTH ORDER SYSTEM

As an additional extension of the method, it was decided to try application to a physical system; a positioning servo utilizing an amplifier, amplidyne and 1/4 H. P. shunt motor. Transfer function were determined experimentally.

And using feedback compensation the system may be overdamped with real roots. Application of the posicast control principle requires that all stored energy be removed when the system reaches zero velocity and the second step is applied with overdamped compensator. In amplidyne system, with output at zero velocity, energy storage is expected in the armature inductance, in the amplidyne quadrature field, and in the amplidyne control field. To dissipate this energy two schemes are attractive; feeding a pulse into the amplifier, and open circuit switching. It was decided to try both, using an analog computer study to guide the techniques.

To provide positive control of the pulse, yet keep its amplitude proportional to the command. This signal was obtained from $\dot{e}_k$. When open circuit switching is used, all three energy storage unit should be opened; however, it did not seem desirable to open the armature circuit of the motor where the energy storage would be quite small. The quadrature field of the amplidyne was available for opening and re-closing with relays and the control field is opened in the process of changing the step input.

of the positioning servo.

Taking open loop frequency response of the whole system and drawing the Bode diagram, it is found that the system has three time constants with one integrator, neglecting values below - 30 db. Fig. 12 (page 40) (point marked x) shows the data taken from the actual system by measuring the ratio of velocity output vs velocity output at lowest frequency (f = .01 cycles).

Assuming open loop transfer function of the system

$$\frac{\theta_c}{E}(S) = \frac{K_v}{S(\tau_1 S+1)(\tau_2 S+1)(\tau_3 S+1)} \qquad (58)$$

$$\frac{\dot{\theta}_c}{E}(S) = \frac{K_v}{(\tau_1 S+1)(\tau_2 S+1)(\tau_3 S+1)} \qquad (59)$$

Taking open loop frequency response of the amplidyne supplying the field current from amplifier, time constant of the motor-load combined was eliminated. Fig. 13 (page 41) shows a schematic diagram for the open loop frequency response test of amplidyne and Fig. 14 (page 42) is the result. Where magnitude ratio was taken as one at the lowest frequency (f = .01 cycles).

From Fig. 14 time constant is found at $\omega = 16.5$ and damping ratio $\varphi = .7$ and amplidyne gain is found as 1000.

Knowing two of the three open loop poles of the system, one time constant in the motor-load combine' 's easily found at W = 8 from Bode diagram in Fig. 12. Fig. 15 (page 43) is the plots for potentiometer calibration (Voltage angular displacement) and Fig. 16 (page 44) is the tachometer calibration curve (voltage induced vs angular velocity).

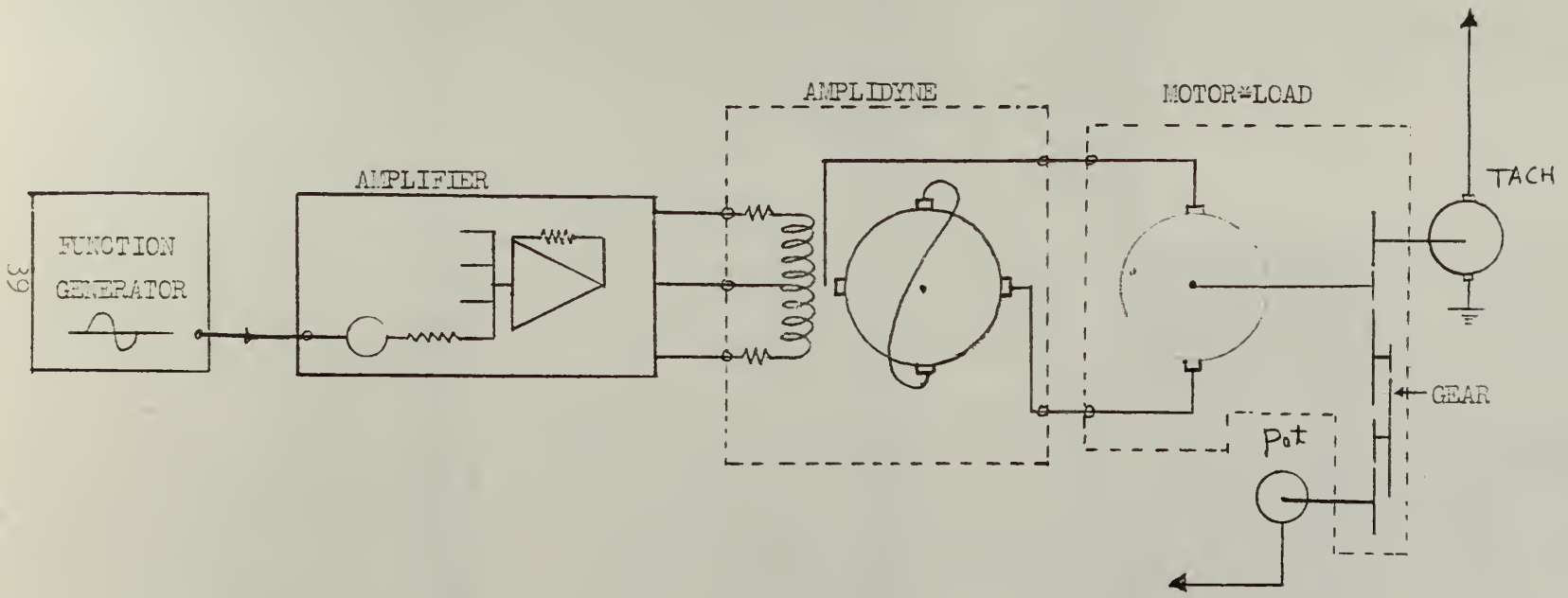


Fig. 11. Schematic Block Diagram of the 4th order Positioning servo.

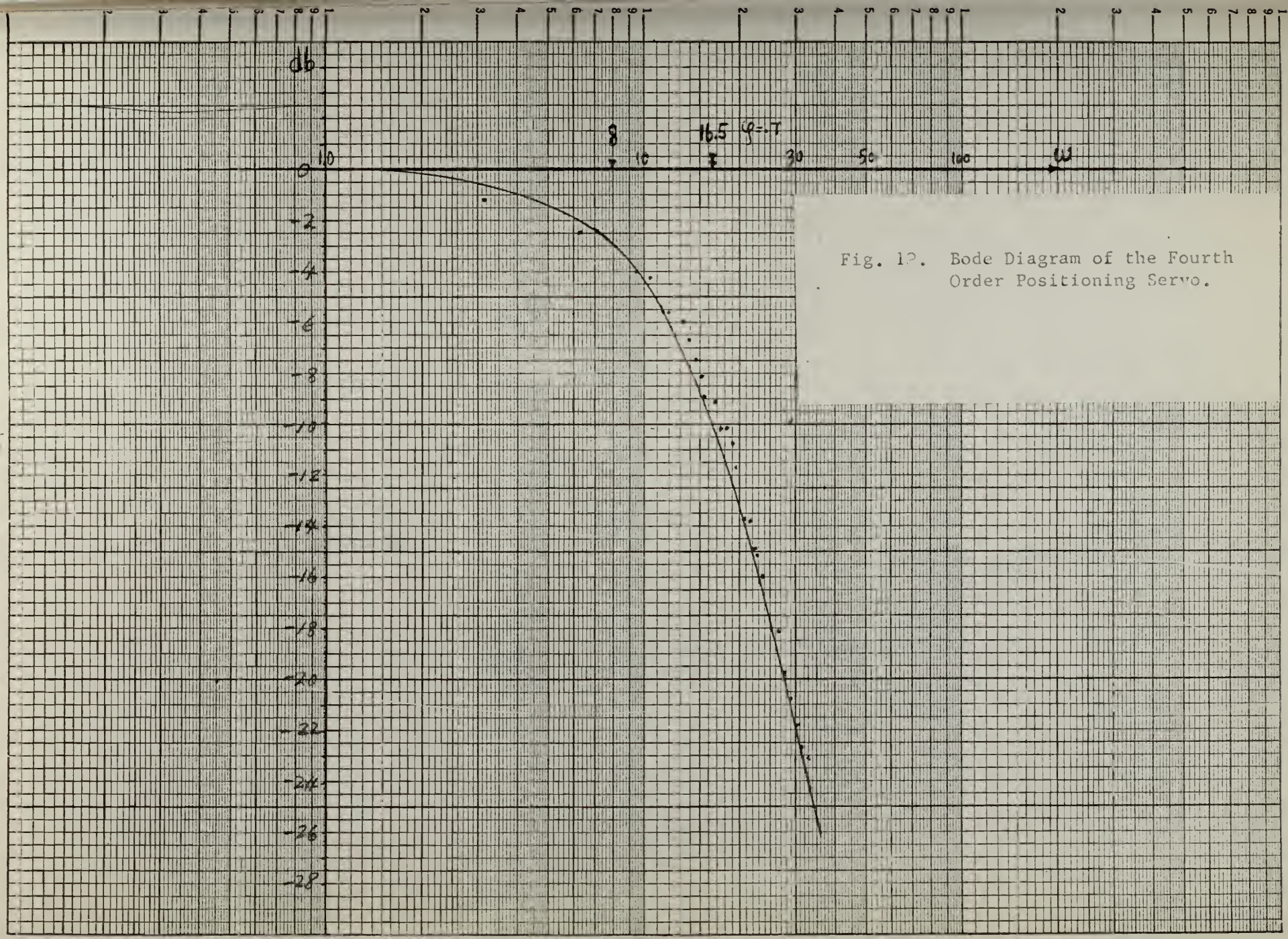


Fig. 19. Bode Diagram of the Fourth Order Positioning Servo.

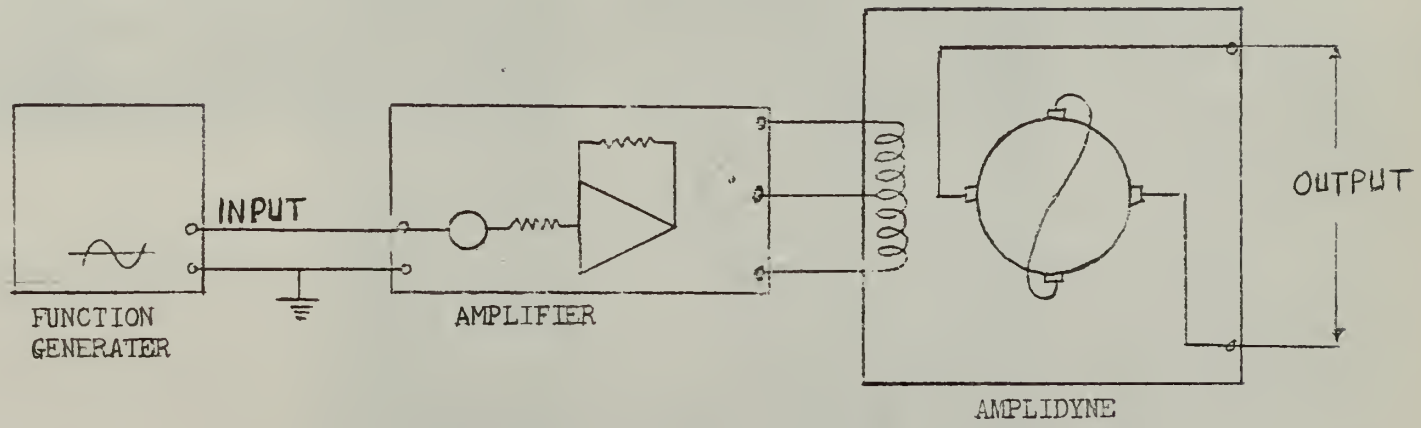


Fig. 13. Schematic Block Diagram for the Open Loop Frequency Response Test of Amplidyne.

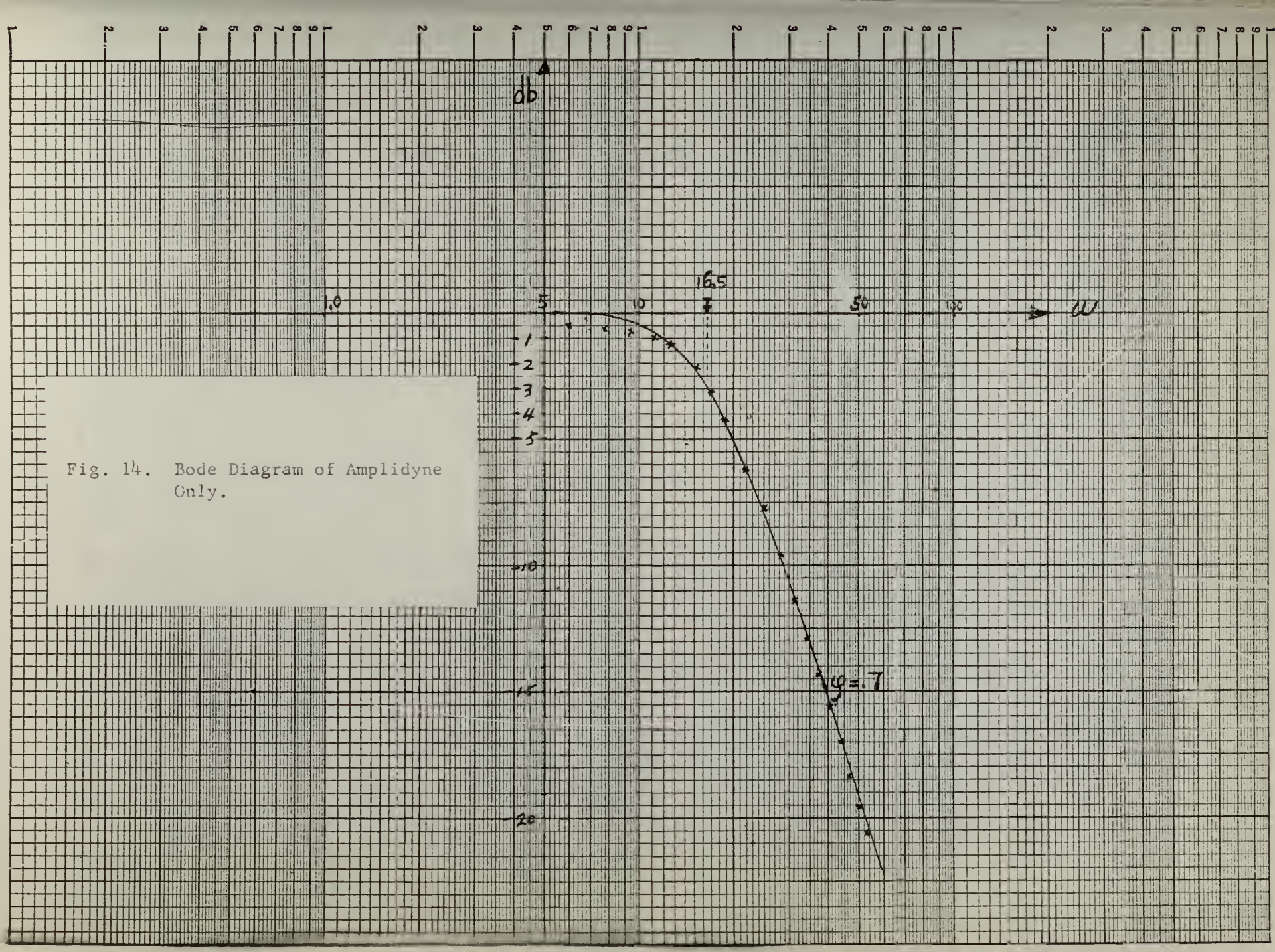


Fig. 14. Bode Diagram of Amplidyne Only.

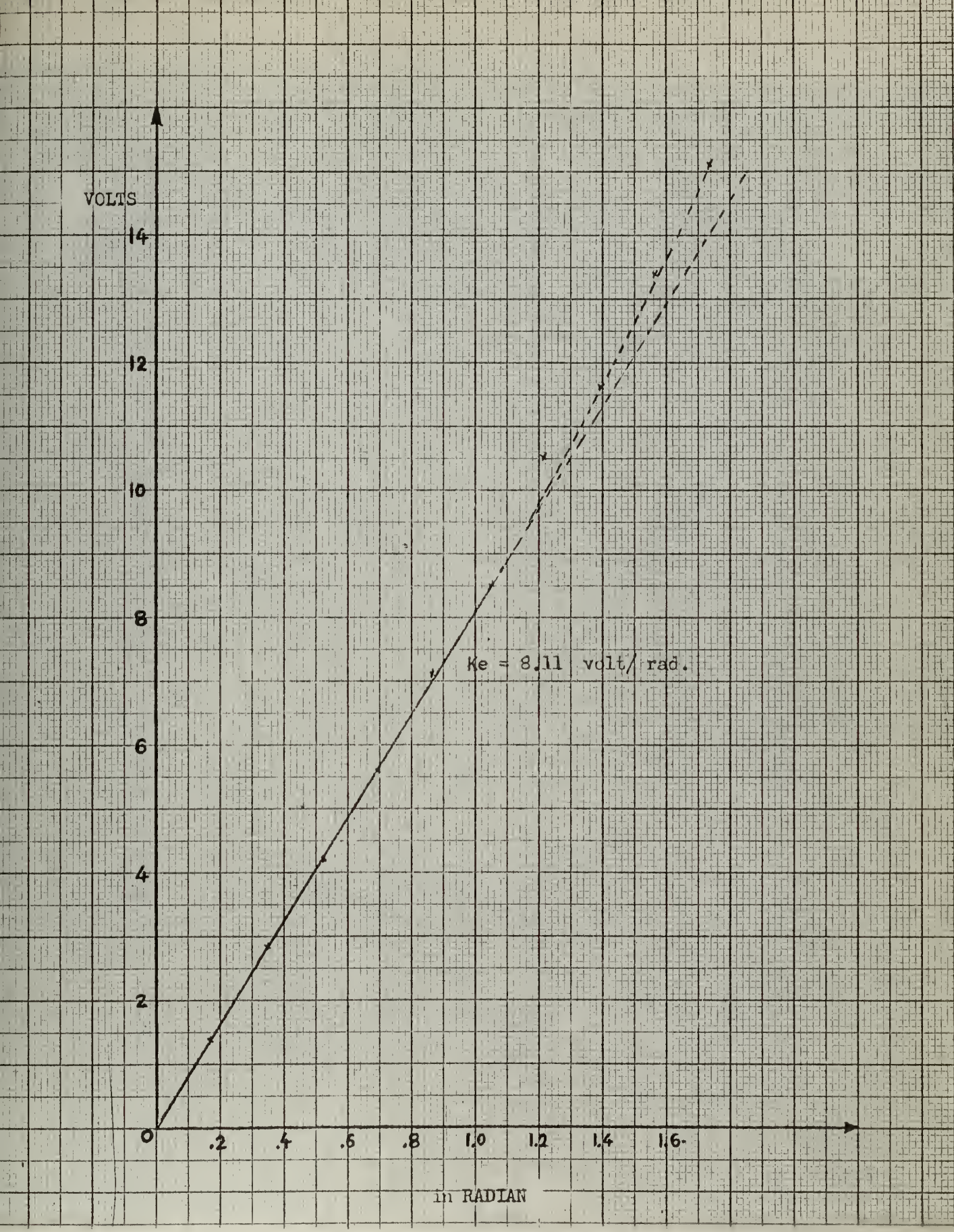


Fig. 15. Pot. Calibration.

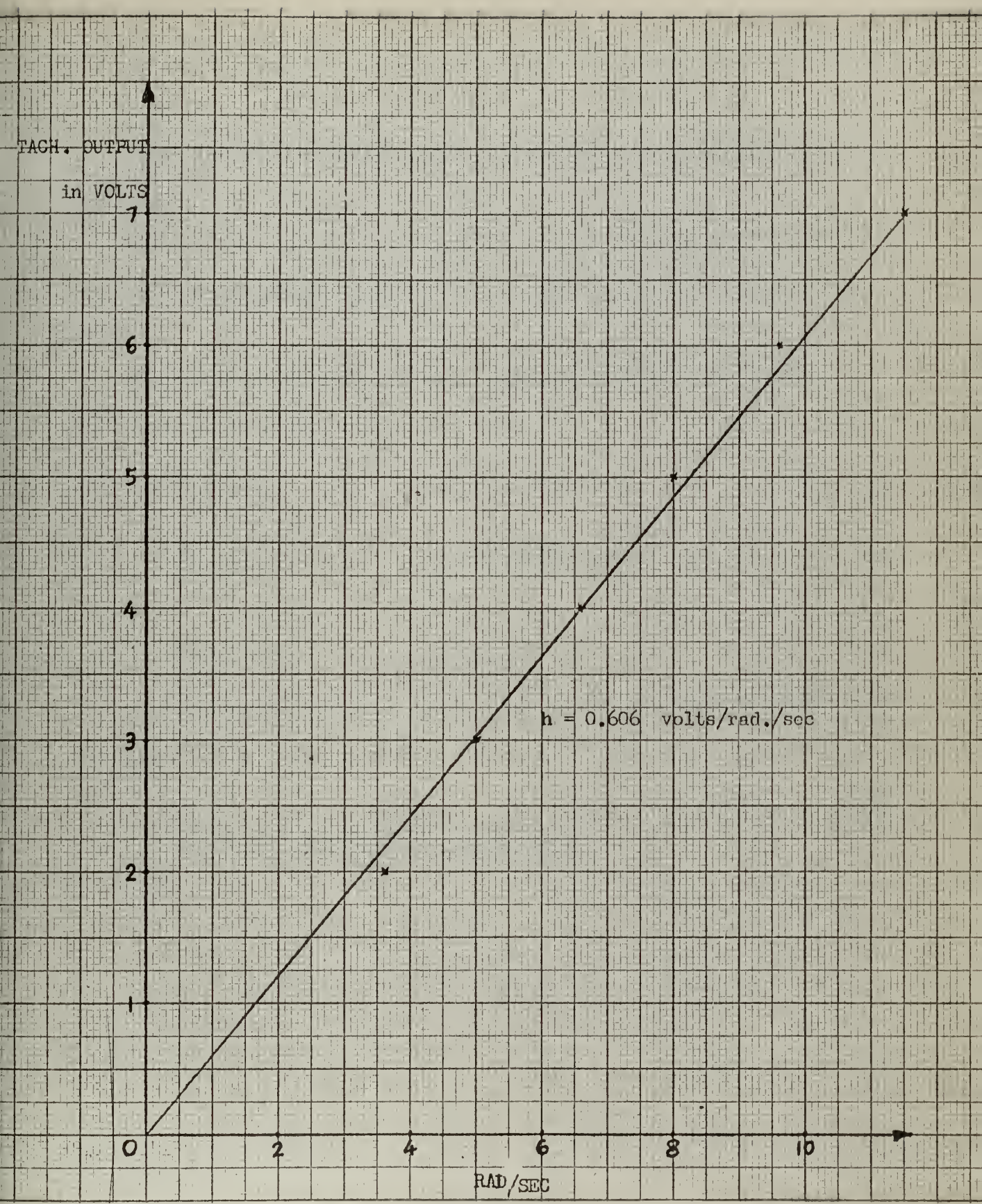


Fig. 16. Tachometer Calibration.

In Fig. 15 slope of the straight line is 0.11 volts/rad. and .606 volts/rad/sec in Fig. 16.

Knowing these two coefficients, the overall system velocity gain is found as .

Drawing the root locus as shown in Fig. 17 (page 46) select the roots of the system underdamped and corresponding gain is found as 24050.

Now, transfer function of each component is predicted closely. Block diagram is shown in Fig. 18a (page 47).

Amplifier : 20 (pure gain)

Amplidyne : $\frac{1000}{S^2 + 23.1S + 272.3}$

Motor-Load : $\frac{1.2}{S(S+8)}$

With the gains indicated (24050) the system is stable but badly underdamped with system open loop function

$$\frac{\theta_c}{E}(S) = \frac{24050}{S(S+8)(S^2+23.1S+272.3)} \qquad (60)$$

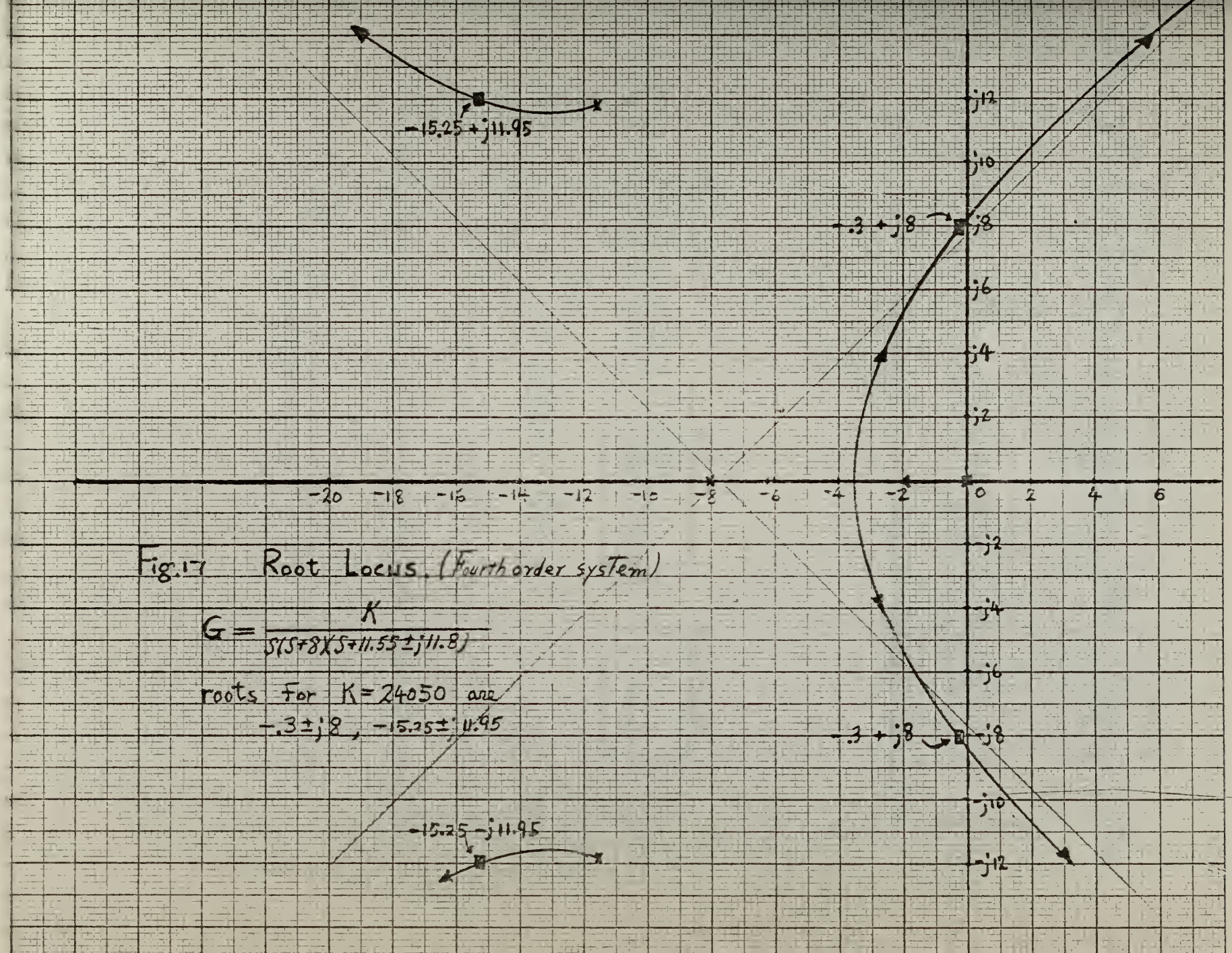
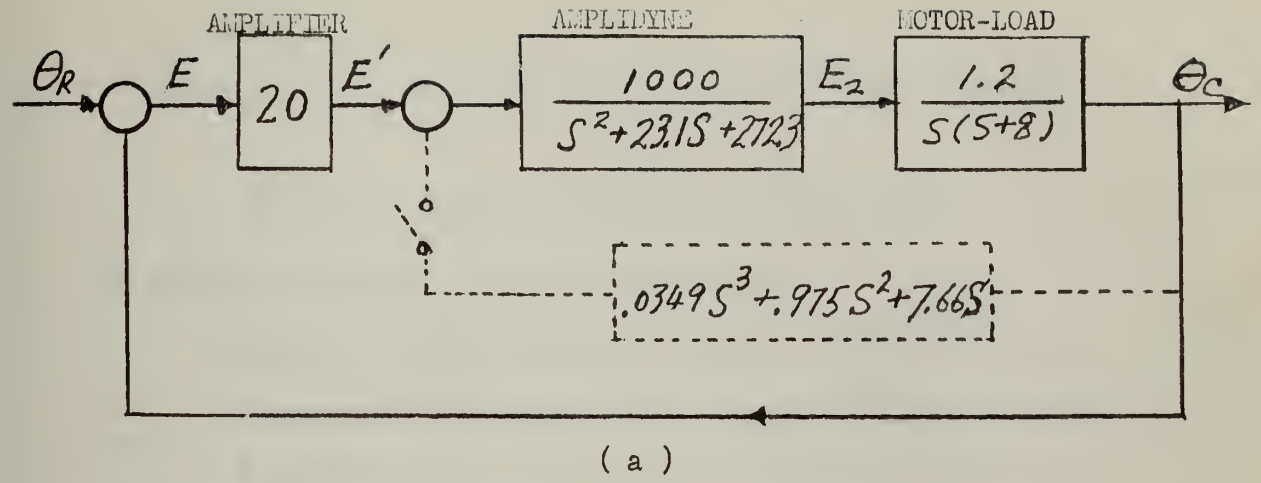


Fig. 1-7 Root Locus. (Fourth order system)

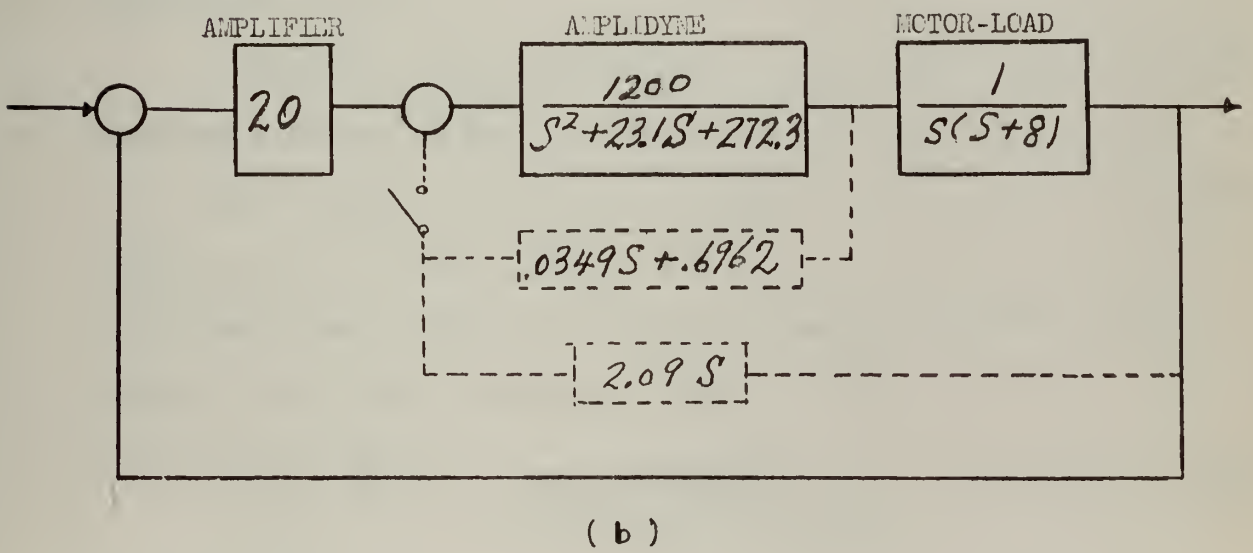
$$G = \frac{K}{S(S+8)(S+11.55 \pm j11.8)}$$

roots For K = 24050 are

$-.3 \pm j8$, $-15.25 \pm j11.95$



(a)



(b)

Fig. 18. Block Diagram of Amplidyne Positioning Servo with Feedback Compensation.

3.3. COMPUTER STUDIES

3.3.1. COMPUTER SET UP

Determined transfer function of the underdamped system is

$$\frac{\Theta_c}{\Theta_R}(s) = \frac{2.40 \times 10^{-3}}{(s+.3 \mp j\,5)(s+15.25 \mp j\,11.75)} \tag{61}$$

and transient response to a step input Au(t), is

$$\theta_c(t) = A - .974Ae^{-.3t}\sin(458^\circ t + 42.5^\circ) + .269Ae^{-15.25t}\sin(\quad\quad) \tag{62}$$

$$\dot{\theta}_c(t) = 7.8Ae^{-.3t}\sin(458^\circ t - 35.35^\circ) + 5.22Ae^{-15.25t}\sin(458^\circ t - 22.\quad) \tag{63}$$

$$\ddot{\theta}_c(t) = -62.5Ae^{-.3t}\sin(458^\circ t - 3.3^\circ) - 101.5Ae^{-15.25t}\sin(458^\circ t - 22.5^\circ) \tag{64}$$

$$\dddot{\theta}_c(t) = 555Ae^{-.3t}\sin(458^\circ t - 3.\quad^\circ) + 1765Ae^{-15.25t}\sin(458^\circ t - 3.\quad^\circ) \tag{65}$$

The maximum overshoot occurs at $t = T_p = \frac{180^\circ + 39^\circ}{458^\circ} = .478$ sec. At this instant $\dot{\theta}_c = o$

$$\theta_c(T_p) = 1.845A = 2A \tag{66}$$

Therefore magnitude of the step is set at

$$A = 1/2 = 1/1.845 = .542 \tag{67}$$

then at $t = T_p$, $\theta_c(T_p) = 1.0$

Now to design the feedback compensation, assuming a suitable transfer function for overdamped operation is known to be

$$G_c = \frac{\Theta_c}{E}(s) = \frac{2405}{s(s+15)(s+20)(s+35)} \tag{68}$$

And system function is

$$\frac{\Theta_c}{\Theta_R}(s) = \frac{2405}{(s+4.08)(s+6.1)(s+26.5)(s+36.0)} \tag{69}$$

It is readily seen that a suitable feedback function converting Equation 60 to Equation 68 and feedback circuit is feeded into next of amplifier which has the gain of 20.

$$H = \frac{20}{24050}\left[41.9S^3 + 1173S^2 + 9222S\right]$$

$$= .0349S^3 + .975S^2 + 7.66S \qquad (70)$$

Fig. 18a (page 47) shows the feedback circuit in dotted line. Rearrange the Equation 60, underdamped system is

$$(S^4 + 31.1S^3 + 457S^2 + 2178S)\,\theta_c(S) = 24050\,E(S) \qquad (71)$$

$$\dddot{\theta}_c(S) = S\,\ddot{\theta}_c(S) = 24050E(S) - 31.1\dddot{\theta}_c(S) - 457\ddot{\theta}_c(S) - 2178\dot{\theta}_c(S) \qquad (72)$$

Where $E = \theta_R - \theta_c$

for overdamped system

$$S\dddot{\theta}_c(S) = 24050E - 730\ddot{\theta}_c(S) - 1630\dot{\theta}_c(S) - 11400\theta_c(S) \qquad (73)$$

from Equations 72 and 73

$$\left[S\dddot{\theta}_c(S)\right]_{\substack{\text{OVERDAMPED}\\ \text{SYSTEM}}} = \left[S\dddot{\theta}_c(S)\right]_{\substack{\text{UNDERDAMPED}\\ \text{SYSTEM}}} - 41.9\dddot{\theta}_c(S) - 1173\ddot{\theta}_c(S) - 9222\dot{\theta}_c(S) \qquad (74)$$

Fig. 19a (page 50) is the computer circuit using pure derivatives. Equation and scaling factors are as follow.

<u>Underdamped system with posicast step input</u>

$$E_1 = \frac{1}{a}\theta_R - \theta_c = .542\,\theta_R - \theta_c$$

$$E_1' = 20E_1 = 10.84\,\theta_R - 20\,\theta_c$$

$$\dddot{\theta}_c = \frac{1}{S}\left(1202.5E_1' - 31.1\dddot{\theta}_c - 457\ddot{\theta}_c - 2178\dot{\theta}_c\right)$$

$$\ddot{\theta}_c = \frac{1}{S}\dddot{\theta}_c, \quad \dot{\theta}_c = \frac{1}{S}\ddot{\theta}_c, \quad \theta_c = \frac{1}{S}\dot{\theta}_c. \qquad (75)$$

<u>Overdamped system with unity step input</u>

$$E = \theta_R - \theta_c$$

$$E' = 20E = 20\,\theta_R - 20\,\theta_c$$

$$\dddot{\theta}_c = \frac{1}{S}\left[1202.5E' - \left(31.1\ddot{\theta}_c + 457\dot{\theta}_c + 2178\theta_c\right) - \left(41.9\ddot{\theta}_c + 1173\dot{\theta}_c + 9222\theta_c\right)\right]$$

$$\ddot{\theta}_c = \frac{1}{S}\dddot{\theta}_c, \quad \dot{\theta}_c = \frac{1}{S}\ddot{\theta}_c \quad \theta_c = \frac{1}{S}\dot{\theta}_c \qquad (76)$$

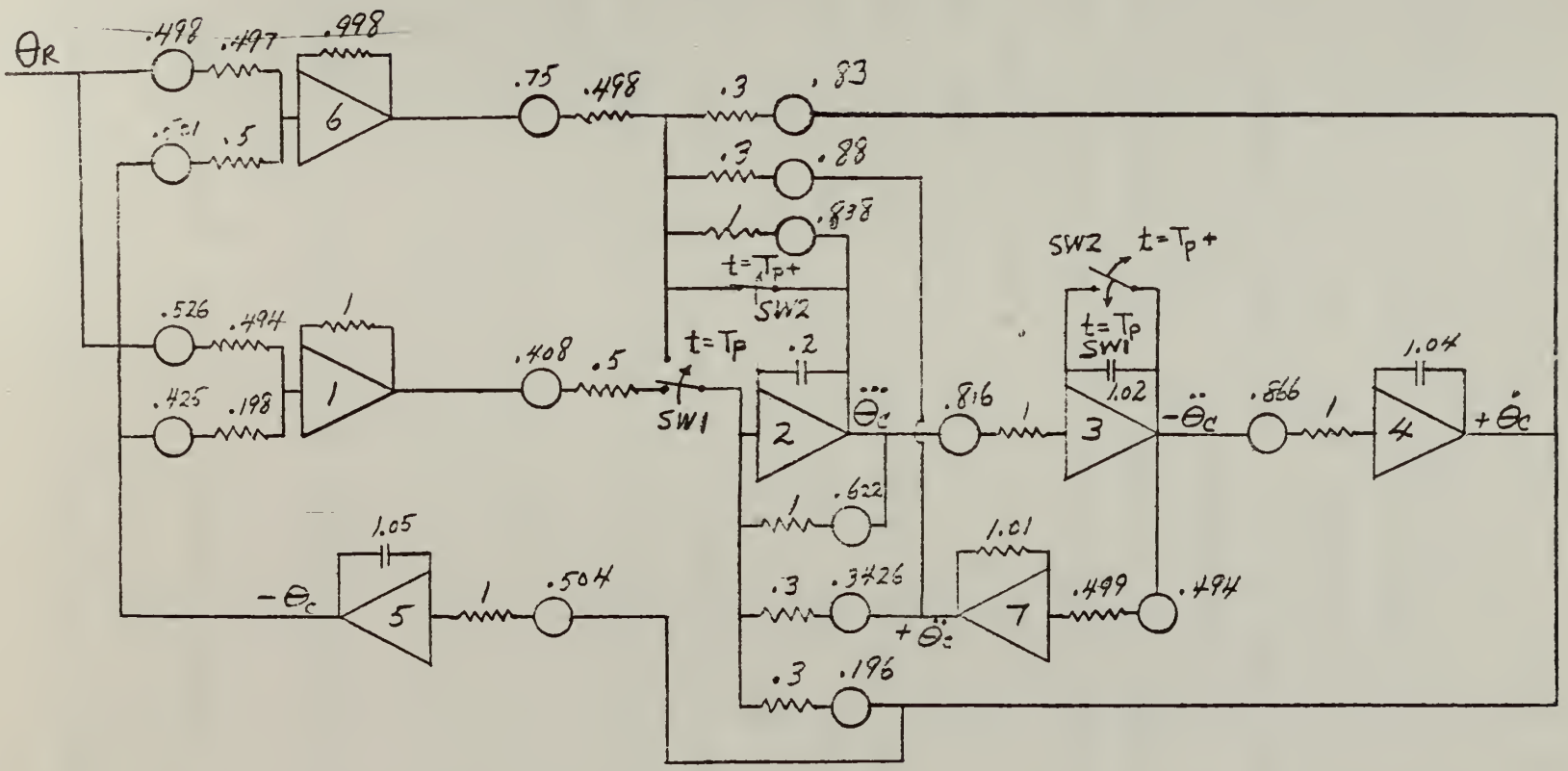


Fig. 19. Computer Circuit and Results for Posicast Control of a Fourth Order System.

A) Computer Circuit. Feedback Compensation, Boundary Conditions are Shorted Out.

Scaling factors are

$$\alpha_t = 10 \quad , \quad \alpha_{\theta_c} = \alpha_{\dot{\theta}_c} = .125 \quad , \quad \alpha_{E_1'} = .1355$$

$$\alpha_{E_1} = .25 \quad \alpha_{\ddot{\theta}_c} = 11.0 \quad , \quad \alpha_{\dot{\theta}_c} = .5 \quad \text{and} \quad \alpha_{\ddot{\theta}_c} = .06$$

In the computer circuit, boundary condition due to acceleration ($\ddot{\theta}_c$) and rate of change of acceleration ($\dddot{\theta}$) is charged at $t = T_p$ in the capacitor of amplifier 3 and 2. This is shorted out by the switch marked SW1, and open again immediately. This switch is controlled utilizing the velocity signal (output of amplifier 4).

Results are shown on Fig. 19B (page 52). Fig. 20A (page 56) shows the computer circuits arranged somewhat similar to the (actual) physical system. And also the boundary conditions are shorted out at $t = T_p$. To make the system overdamped, additional $\dot{\theta}_c$, $\ddot{\theta}_c$, $\dddot{\theta}_c$ signal feedback circuit is necessary in the underdamped circuits. But pure higher order derivatives signals are not available except velocity signal. Therefore, feedback circuit is different from Fig. 19A. In Fig. 18b (page 47) assuming motor-load gain as one and amplidyne gain as 1202.5 and E_2 is designated as the output of the amplidyne (or input to the motor armature). Then

$$E_2 = S(S+8)\theta_c = \ddot{\theta}_c + 8\ddot{\theta}_c \tag{77}$$

$$\dot{E}_2 = SE_2 = \dddot{\theta}_c + 8\ddot{\theta}_c \tag{78}$$

From Equations 70, 77 and 78 required feedback signals can be expressed with E_2, $\dot{E}_2$ and $\dot{\theta}_c$

$$.0349\dddot{\theta}_c + .915\ddot{\theta}_c + 7.66\dot{\theta}_c = .0349\dot{E}_2 + .696E_2 + 2.19\dot{\theta}_c \tag{79}$$

Where different arrangement of the gains in motor-load and amplidyne does not change the overall system characteristic nor analysis of the

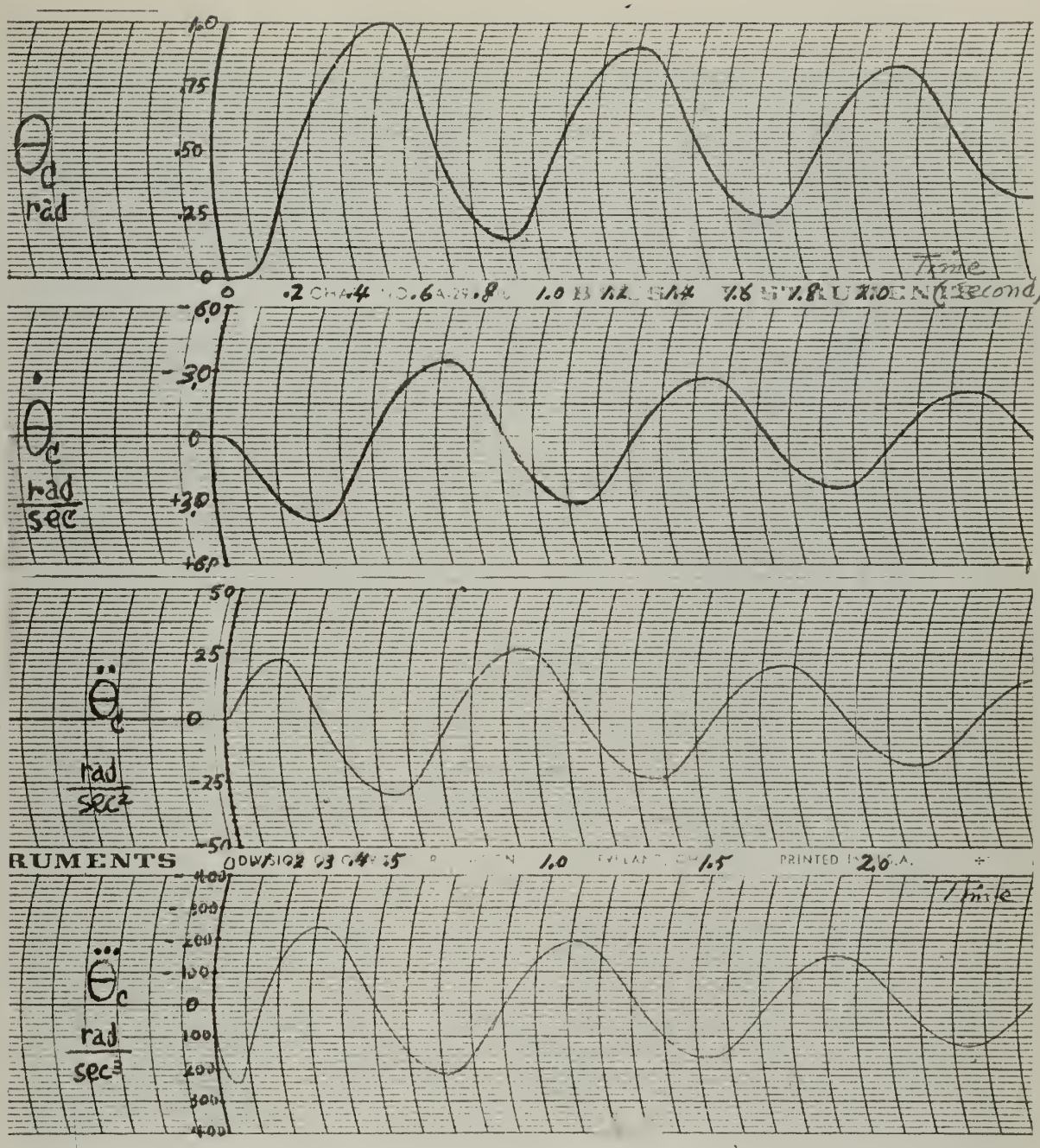


Fig. 19 B. Results of Computer Studies.

a) Underdamped.

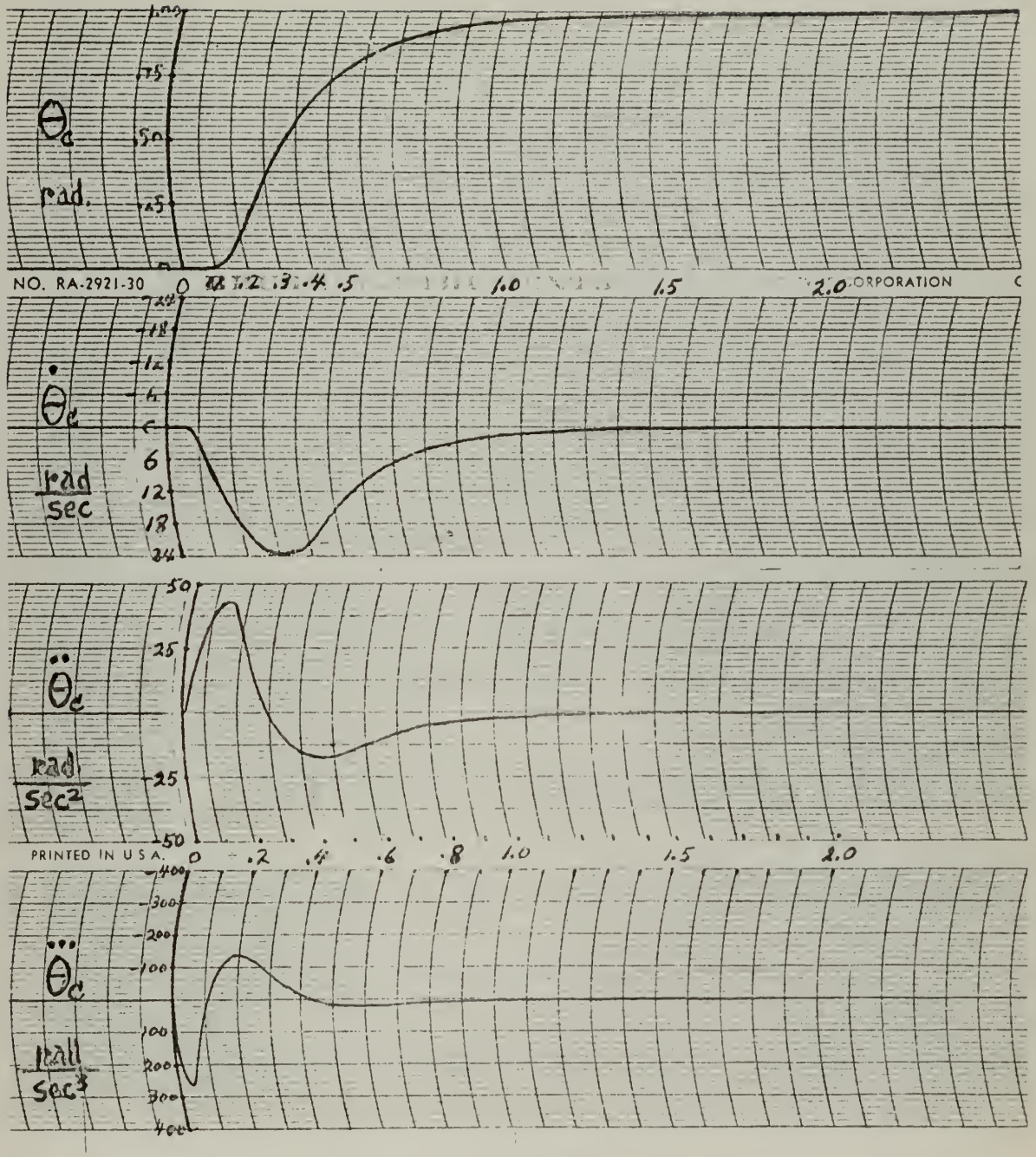


Fig. 19 B. Results of Computer Studies.

b) Overdamped.

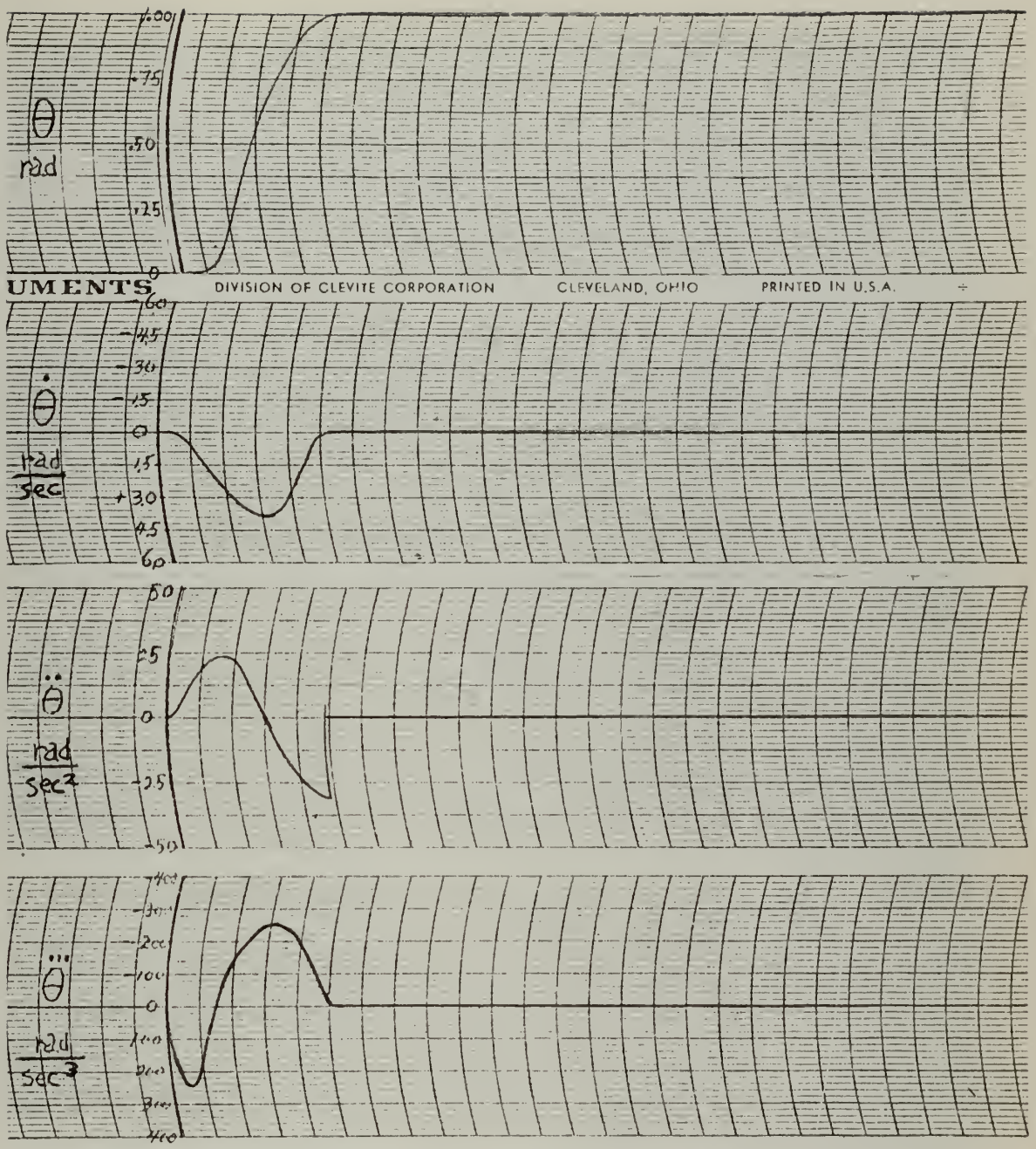


Fig. 19 B. Results of Computer Studies.

c) Final Result.

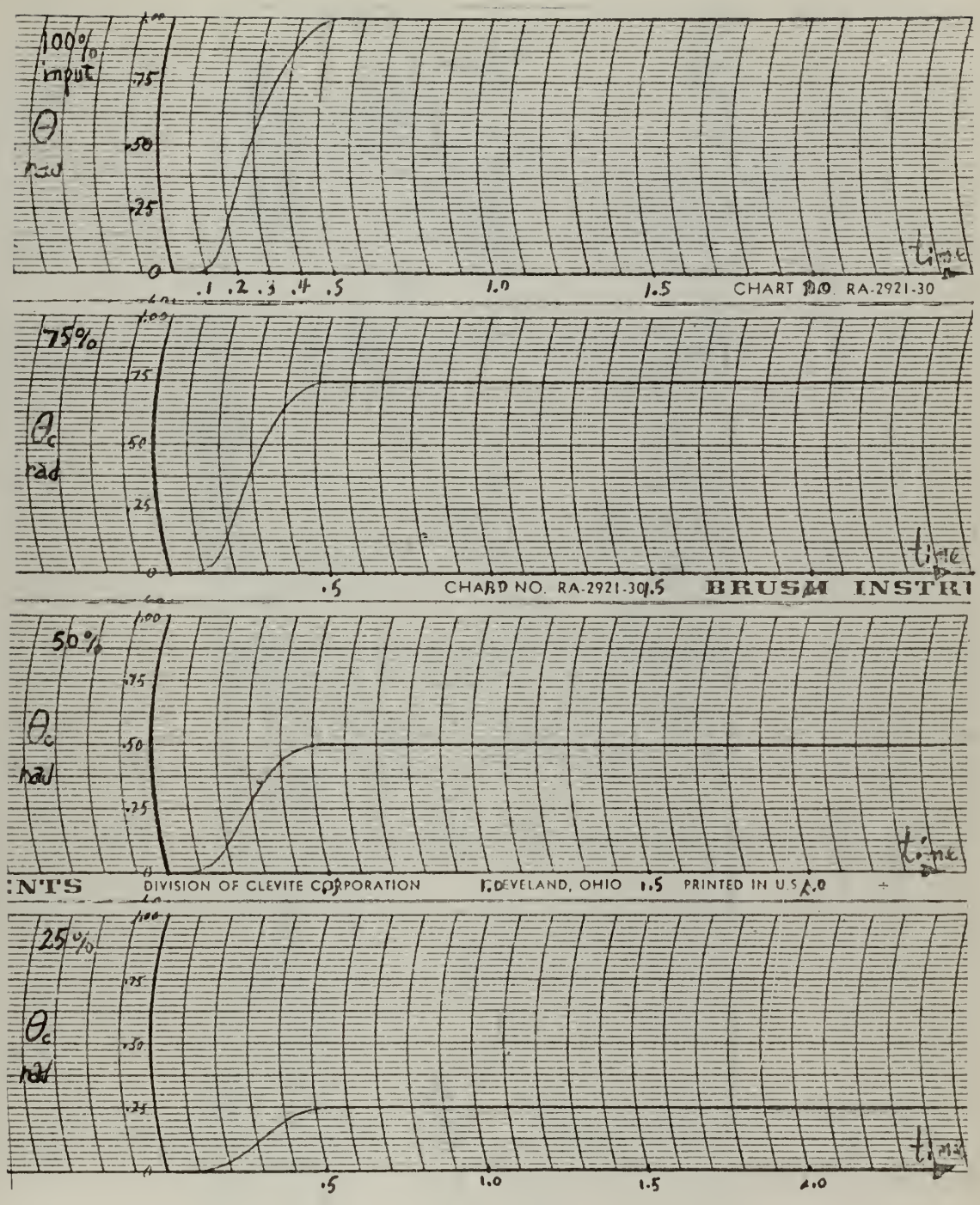


Fig. 19 B . Result of computer studies.

d) Step responses with different magnitudes of the command.

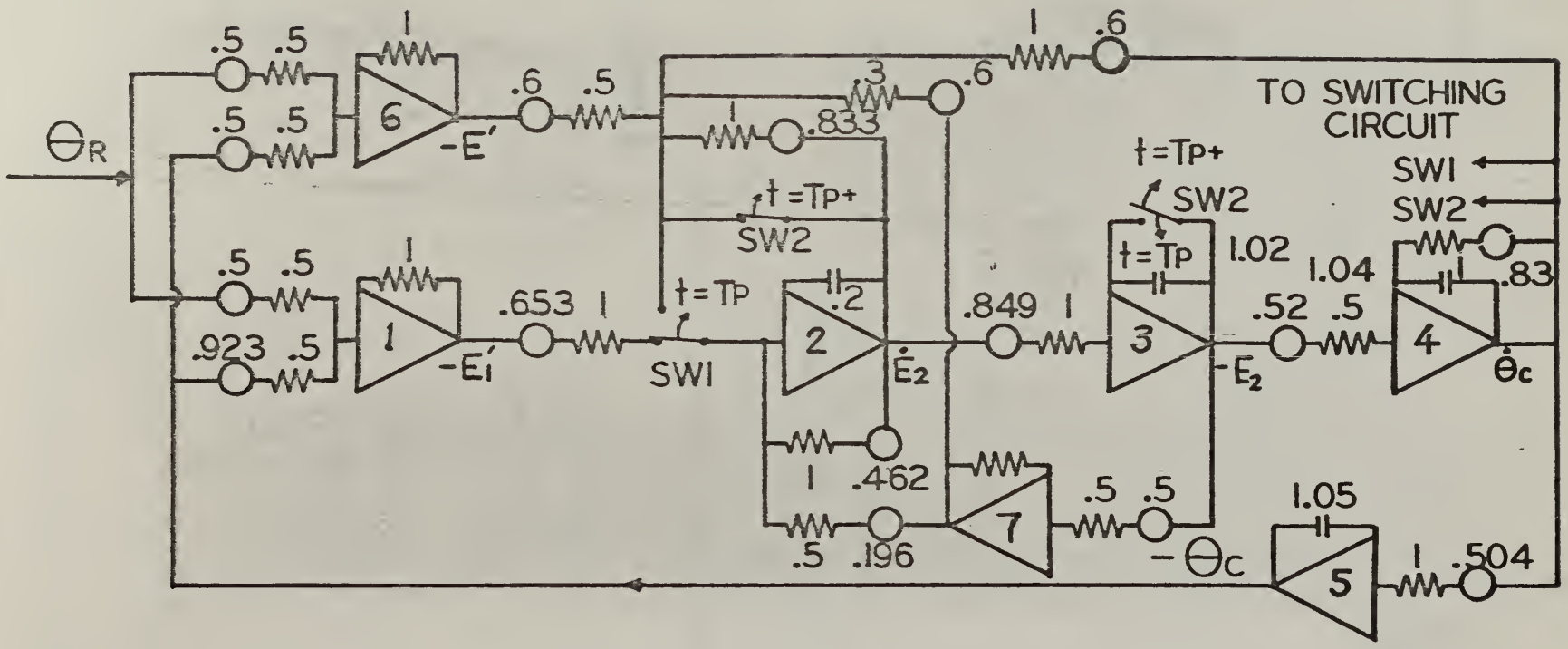


Fig. 20. Computer Circuit and Results for Posicast Control of a Fourth Order System.

A) Computer Circuit Using Shorting Switches to Simulate Opening the Inductive Circuit.

of the problem, block diagram is shown in Fig. 19B (page 51), and equations and scaling factors are as follows.

Underdamped system

$$E_1 = .542\theta_R - \theta_c$$
$$E' = 20E_1 = 10.84\theta_R - 20\theta_c$$
$$\dot{E}_2 = (1/s)(1202.5E' - 23.1\dot{E}_2 - 272.3E_2)$$
$$E_2 = (1/s)\dot{E}_2 \quad , \quad \dot{\theta}_c = (1/s+8)E_2 \quad , \quad \theta_c = (1/s)\dot{\theta}_c \qquad (80)$$

Overdamped system

$$E = \theta_R - \theta_c$$
$$E' = 20E = 20\theta_R - 20\theta_c$$
$$E'' = 20\theta_R - \theta_c - (.034\dot{E}_2 + .696E_2 + 2.04\dot{\theta}_c)$$
$$\dot{E}_2 = \frac{1}{s}(1202.5E'' - 23.1\dot{E}_2 - 272.3E_2)$$
$$= \frac{1}{s}\left[1202.5E' - (23.1\dot{E}_2 + 272.3E_2) - (41.9\dot{E}_2 + 831E_2 + 2520\dot{\theta}_c)\right] \qquad (81)$$

Scaling factors are the same as before with the addition of $\alpha_{E_2} = .6$ and $\alpha_{\dot{E}_2} = 5$. And results are presented on Fig. 20B (page 58).

3.3.2. PULSE TECHNIQUE

In Fig. 21A (page 63) pulse is applied before the second posicast step is connected to the system and removed with the first posicast input.

Computer circuit is exactly the same as Fig. 20A (page 56) except the branch in which pulse is applied. For the higher order system, transient is continually varying after second step is connected to the system as shown on Fig. 20 Bb (page 58).

In order to eliminate this variation, the right amount of transient with opposite polarity may do the job.

The switch is operated at velocity equals zero changing the input and inserting damping. An instant after the switch is operated, the

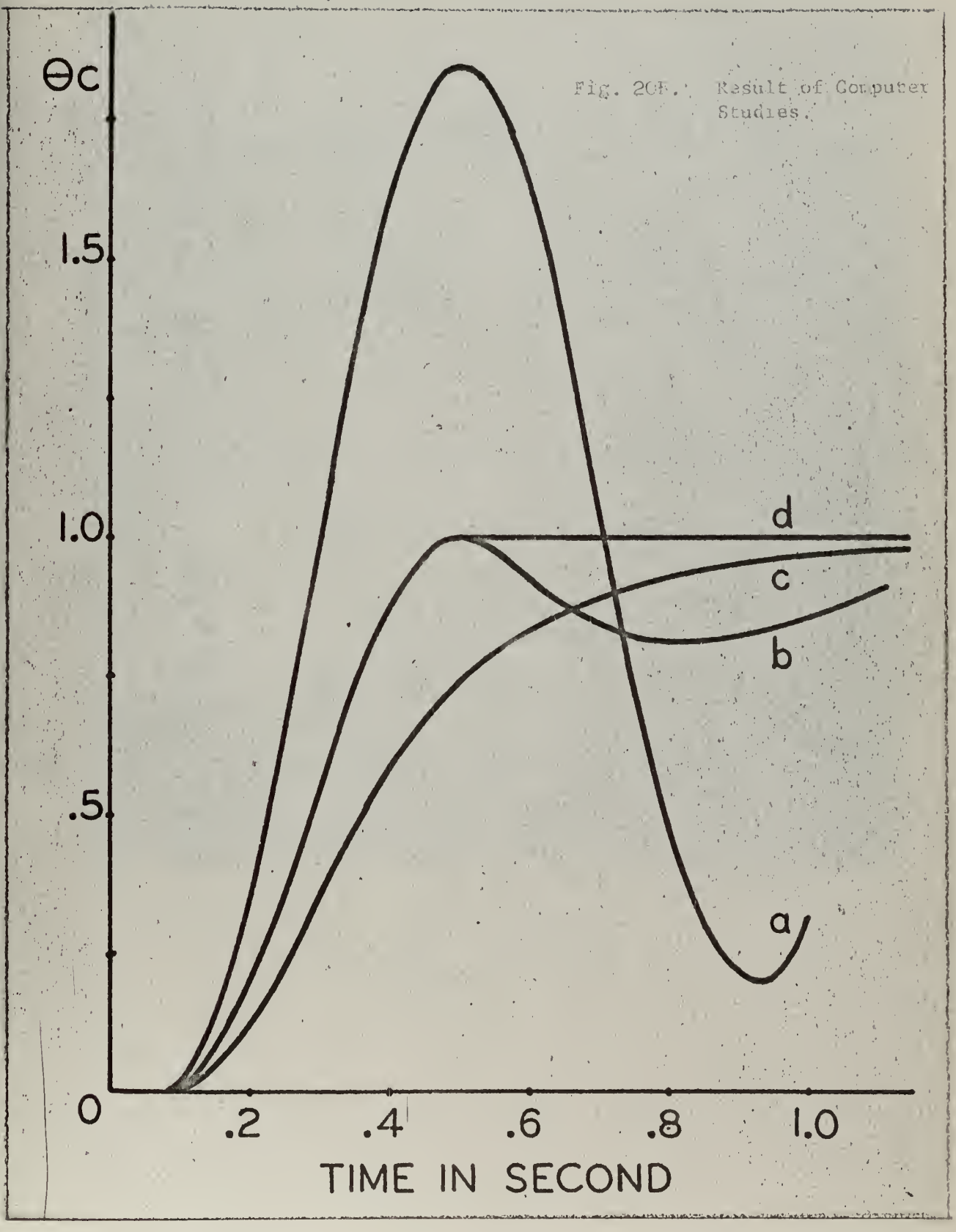


Fig. 20F. Result of Computer Studies.

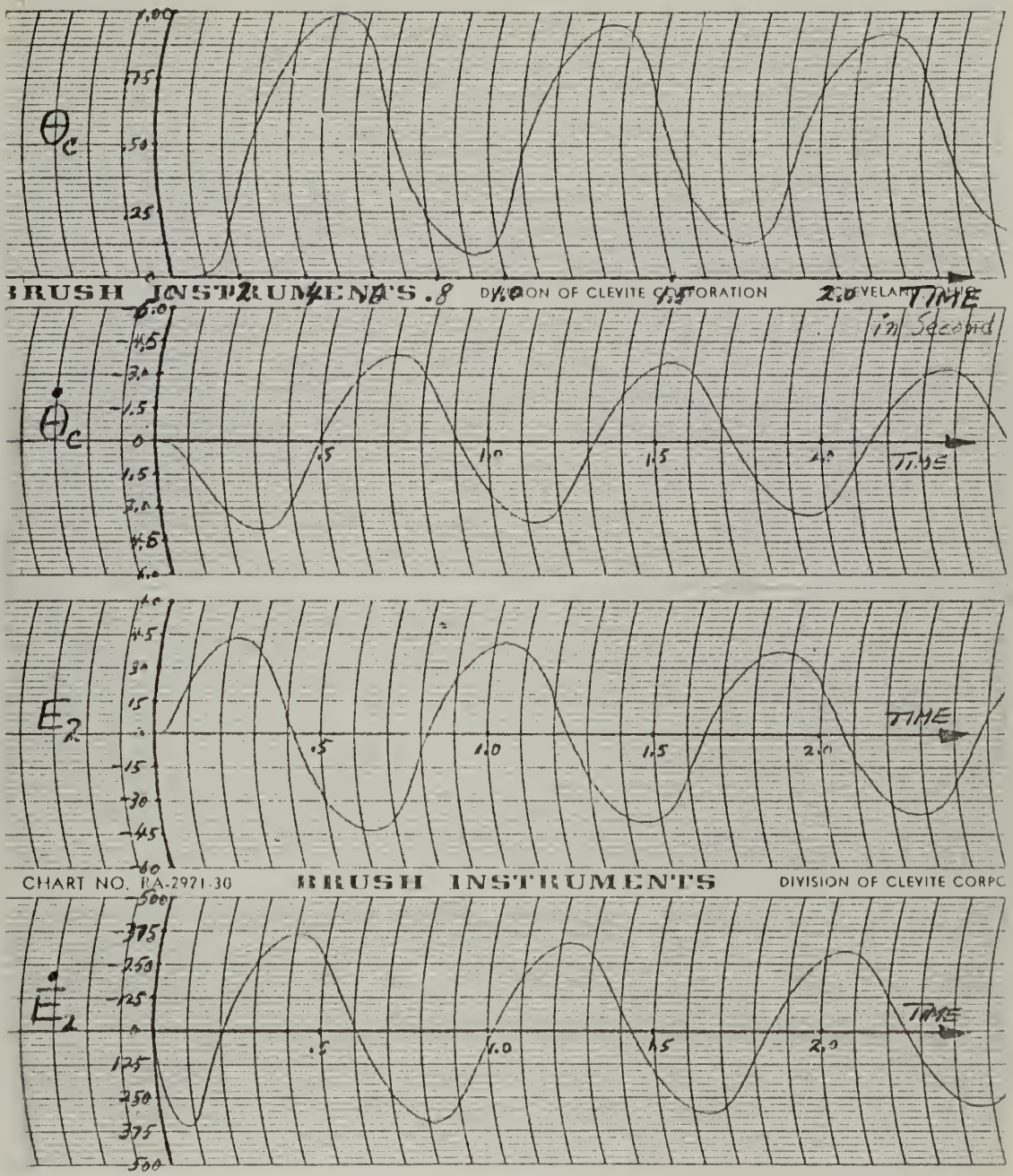


Fig. 20 B. Result of Computer Studies.

a) Underdamped.

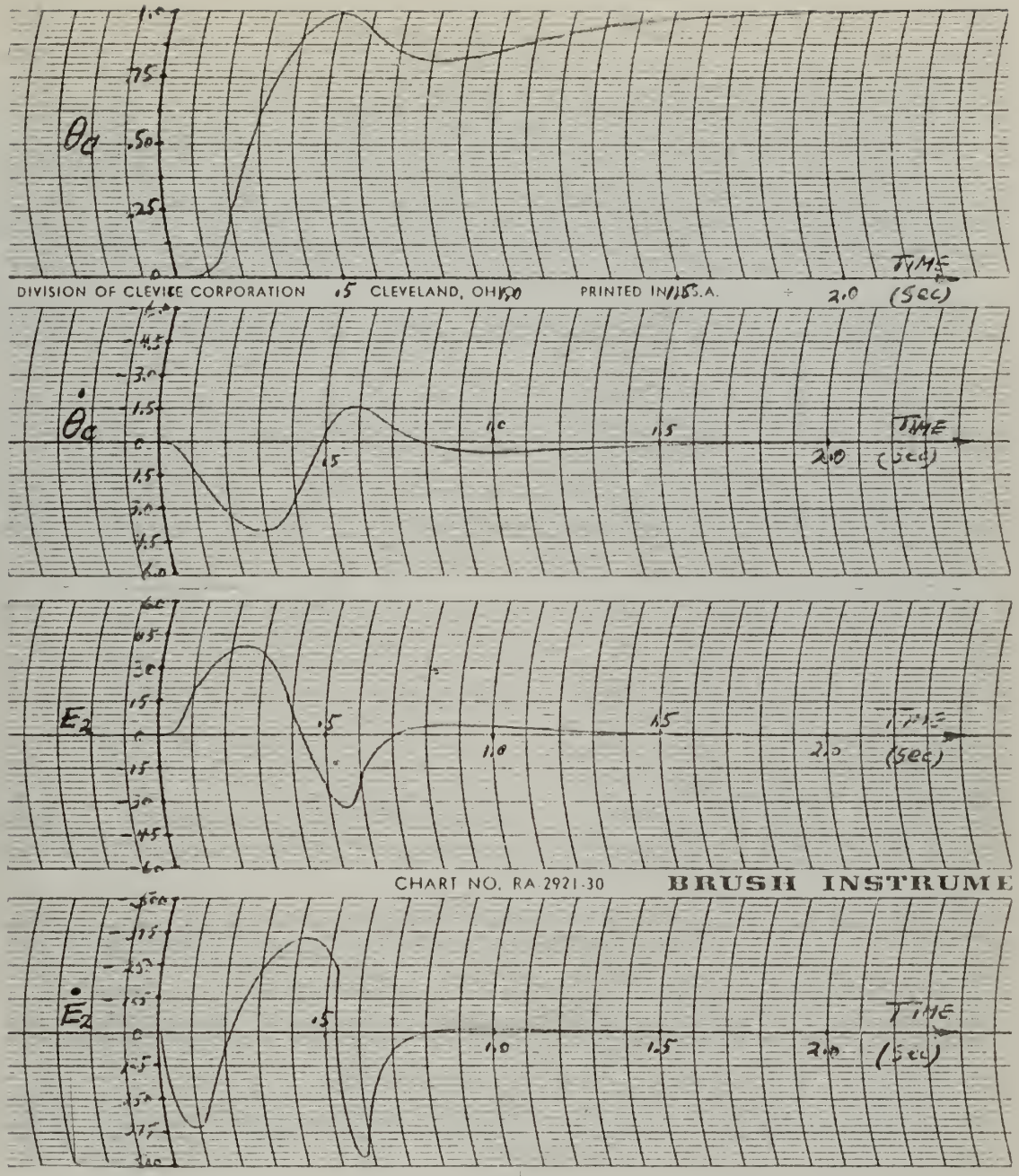


Fig. 20 B. Result of computer studies.

b) Posicast with overdamping circuit but without cancelling stored energy.

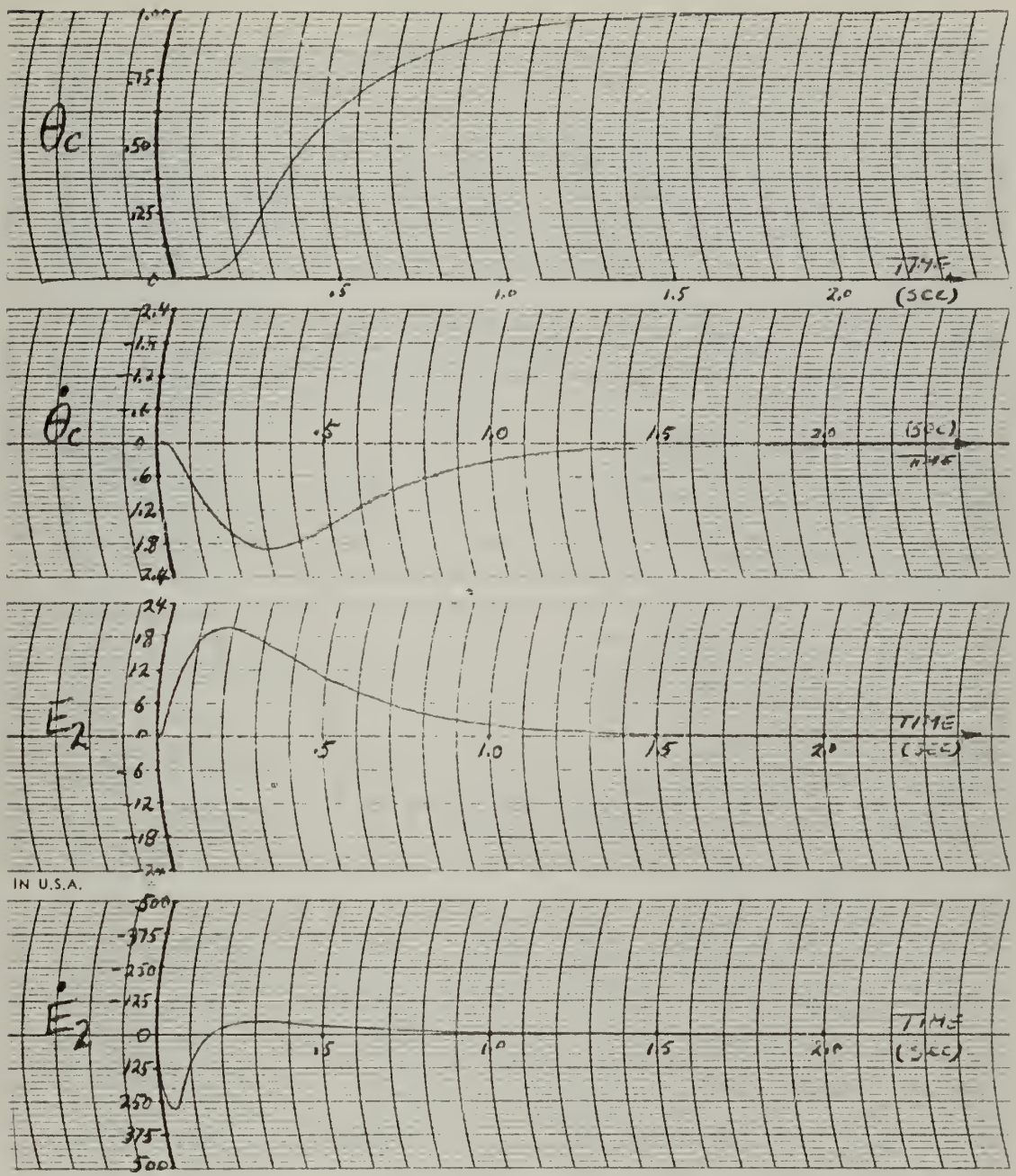


Fig. 20 B. Result of computer studies.

c) Overdamped system response.

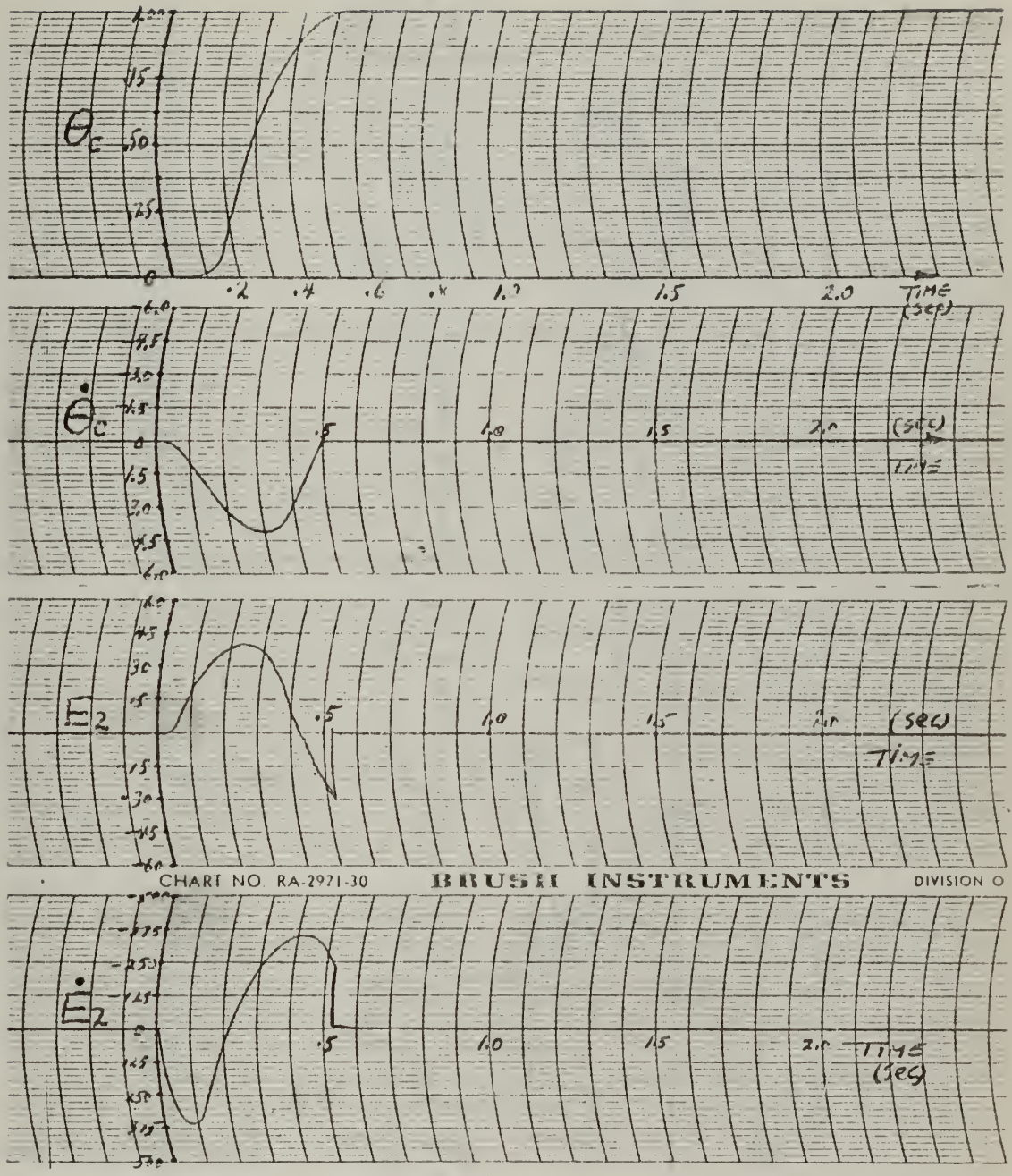


Fig. 20 B. Result of computer studies.

d) Final result with short circuits.

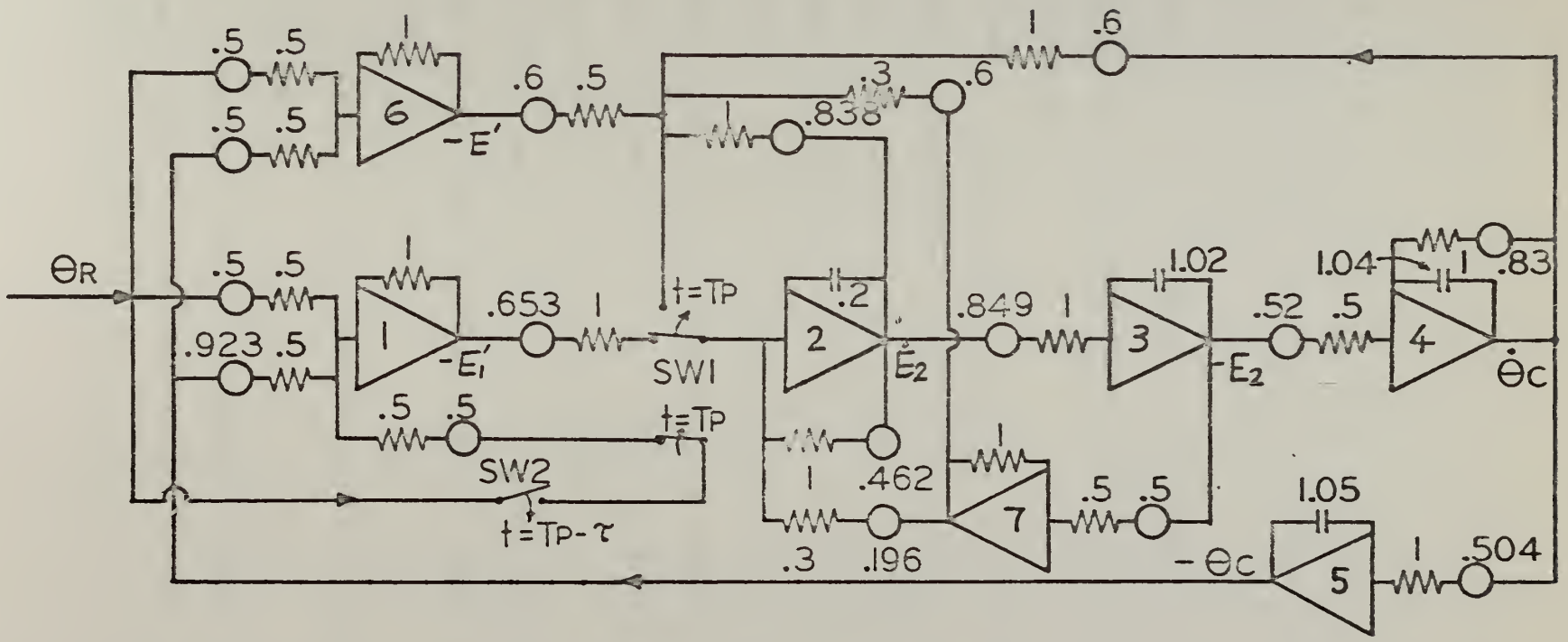


Fig. 21. Computer Circuit and Results for Posicast Control of a Fourth Order System.

A) Computer Circuit. Using a Pulse Input to Counteract the Stored Energy.

decreasing displacement transient due to the stored energy is similar to that at the starting point of the overdamped system response.

If the switch is operated to change the input, but no damping is added, the transient after first peak of overshoot is closely estimated as $-\theta_c(t')\,u(t')$, where $t' = t - T_p$.

Therefore, a pulse is chosen to counterbalance the transient due to stored energy, but in applying this pulse the time lags in the system must be considered.

The time of the pulse width is closely related to the time lag of the system and the magnitude of the pulse is chosen near the initial value of the system error for convenience. In actual computer studies, these values are determined by trial and error. And it is certain that the magnitude of the pulse should be greater than $(1 - 1/n)$, and with greater magnitudes, a shorter pulse width is needed.

Choosing the magnitude of the pulse, the system response could be deadbeat with only one value of pulse width.

The pulse is applied before the second step; it is applied to the underdamped system, and disconnected when the second step and overdamped circuit are connected.

Fig. 21B (page 65) shows the final result of deadbeat response applying the pulse.

Fig. 21C (page 66) shows the deadbeat response applying wider and smaller pulse than in Fig. 21B and compares it with that obtained varying the magnitude of the pulse and pulse width.

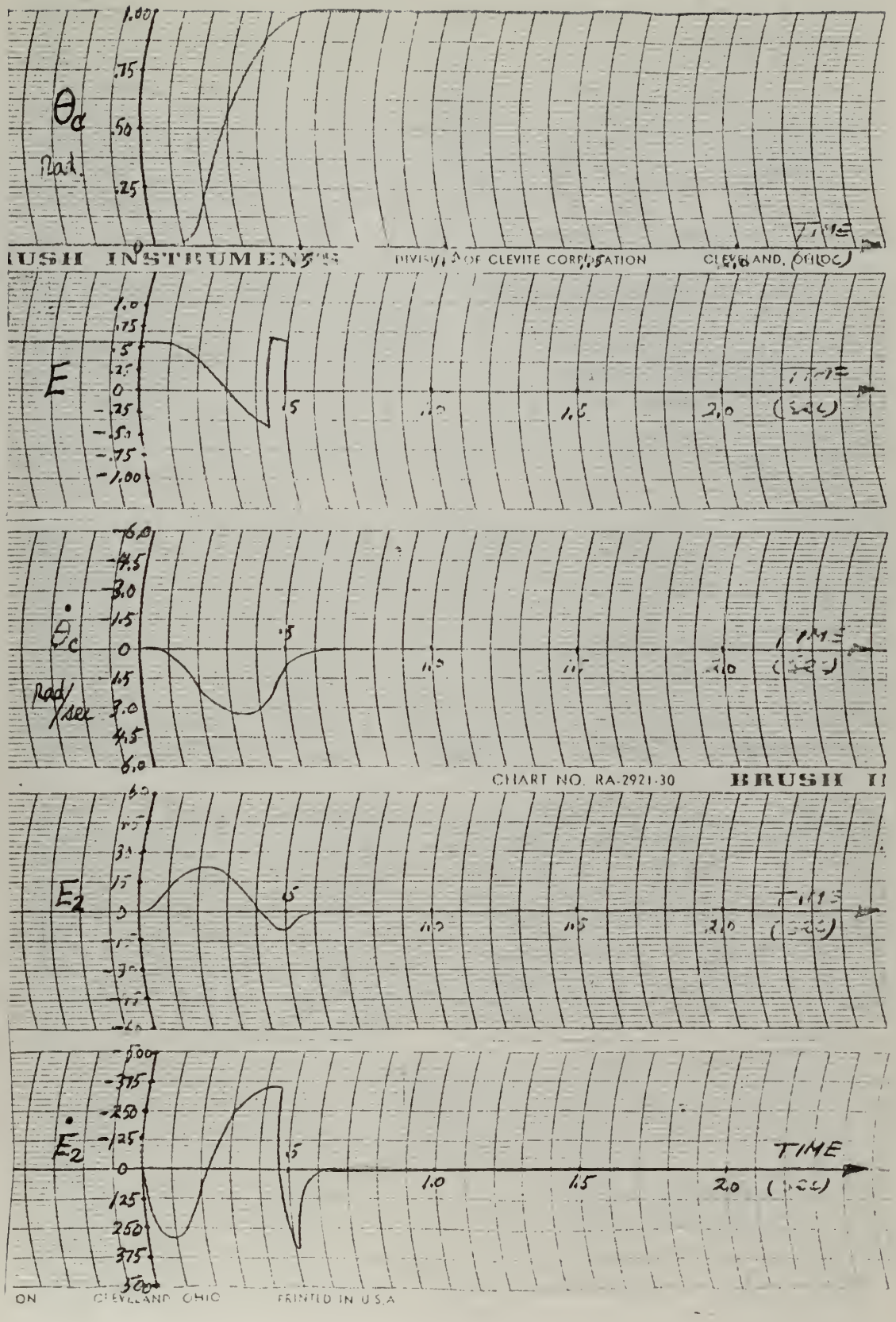


Fig. 21 B. Final result of computer studies with pulse.

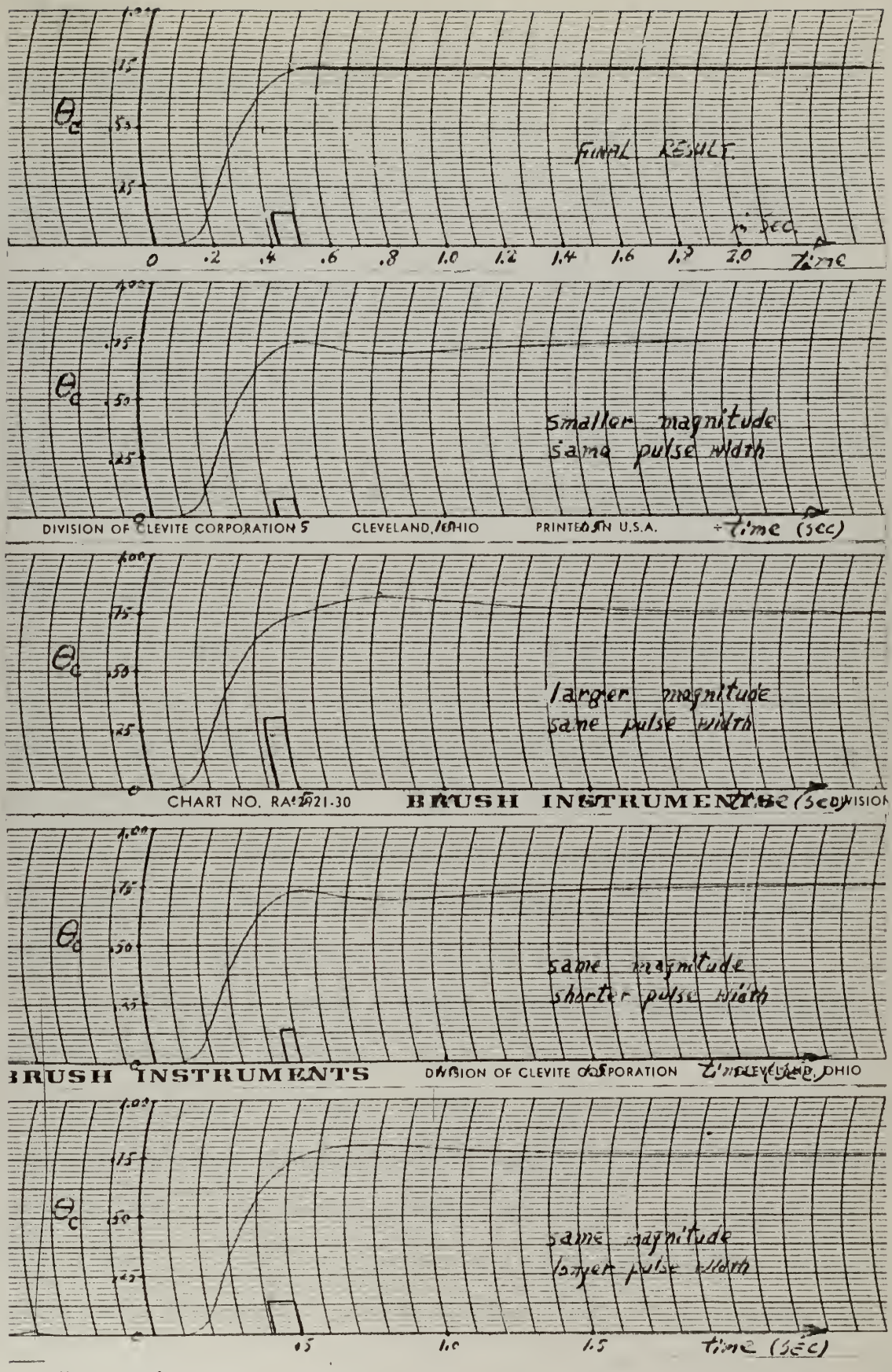


Fig. 21C. Result of computer studies with varying magnitude of the pulse and pulse width.

3.4. PHYSICAL SYSTEM STUDIES

Fig. 22a (page 69) and Fig. 22b (page 70) shows the circuit diagrams for the positioning servo with open circuit switching technique and pulse injection techniques.

Note that in each case only velocity and acceleration feedback were used, since noise prevented addition of a $\ddot{\theta}_o$ channel.

As a result the compensated system had two real and two complex roots instead of four real roots. However, the real roots were dominant and the compensated system had an overdamped step response.

Fig. 23 (page 71) shows a system response for underdamped and overdamped cases.

Fig. 24 (page 73) shows a typical transient response obtained when the quadrature field was opened and compares it with that obtained using posicast control but without opening the quadrature field. Note that the characteristic dip obtained when the energy is not cancelled was predicted by the computer study of Fig. 20 Bb (page 58) but in this case small amplitude oscillations due to the complex roots are apparent. Similar results were obtained for various magnitudes of step.

Fig. 25 (page 74) shows a family of step responses opening the quadrature field of amplidyne.

Fig. 26 (page 76) shows a family of step responses using the pulse injection technique with carefully adjusted pulse width. Note that the responses are all essentially deadbeat as desired. For large input steps there is a slight oscillation, and it is presumed that saturation in the magnetic circuits has prevented proportional energy storage, so that the pulse width (τ) as adjusted for normal steps is too wide for larger steps.

Adjustment of the pulse width (τ) is fairly critical. Fig. 27 (page 77) shows the effect of variations in pulse width (τ) on the response.

Note that the pulse must be applied somewhat before the second posicast step to compensate for the time lags in the system.

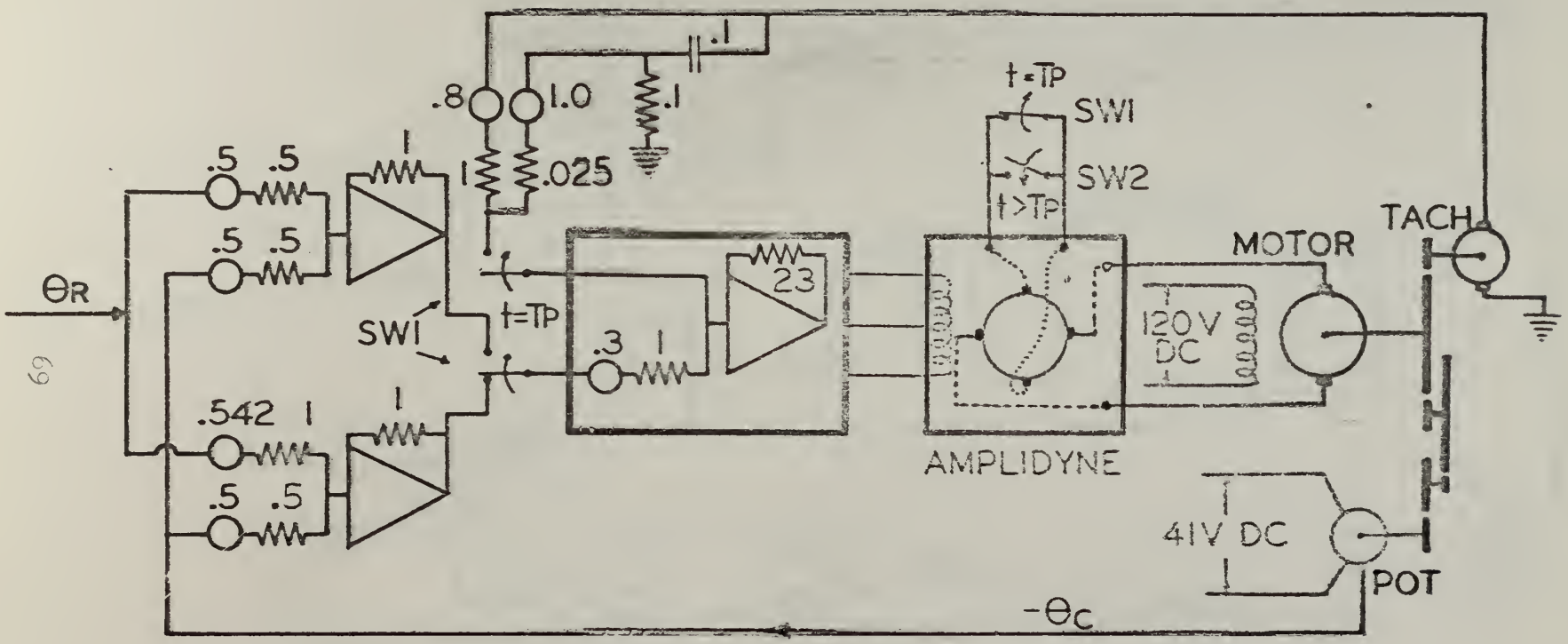


Fig. 22. Circuit Diagrams for Modified Posicast Control of Positioning Servo.

a) With Open Circuit Switching to Dissipate Stored Energy.

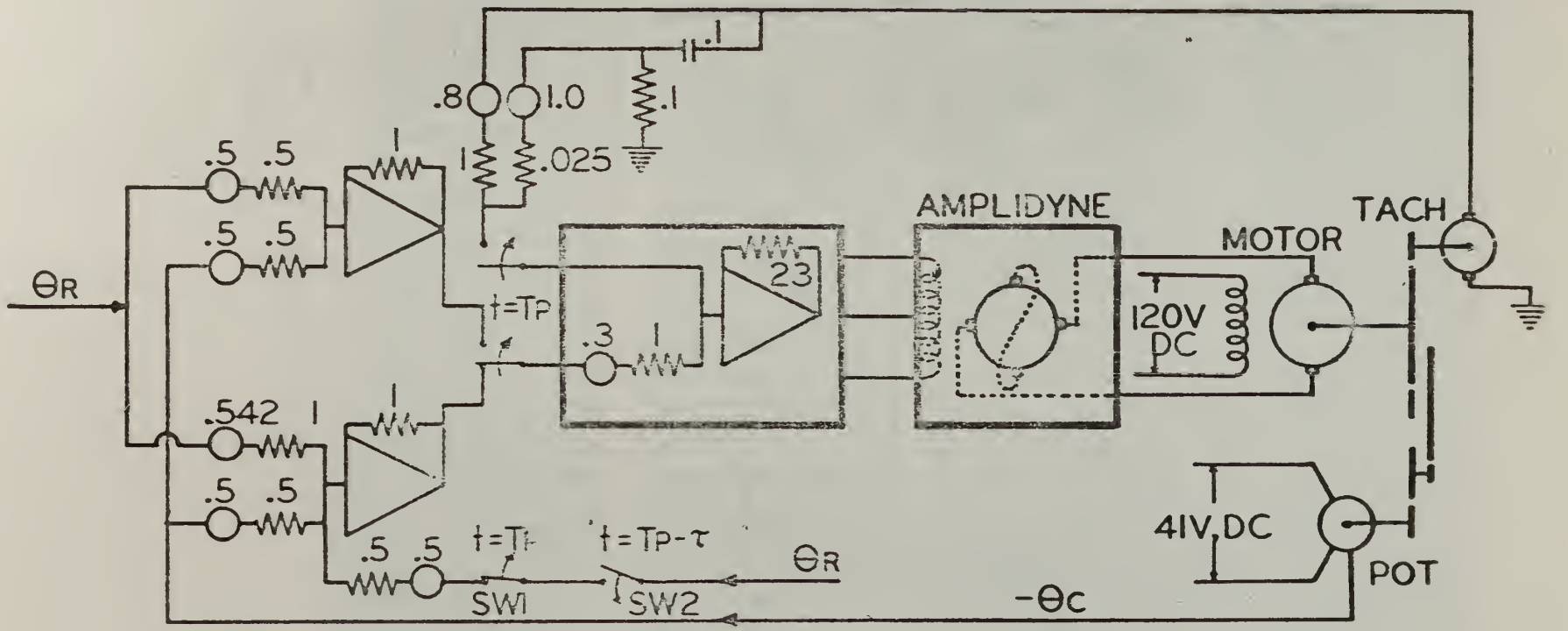


Fig. 22. Circuit Diagrams for Modified Posicast Control of Positioning Servo.

b) With Pulse Injection to Cancel Transient Due to Stored Energy.

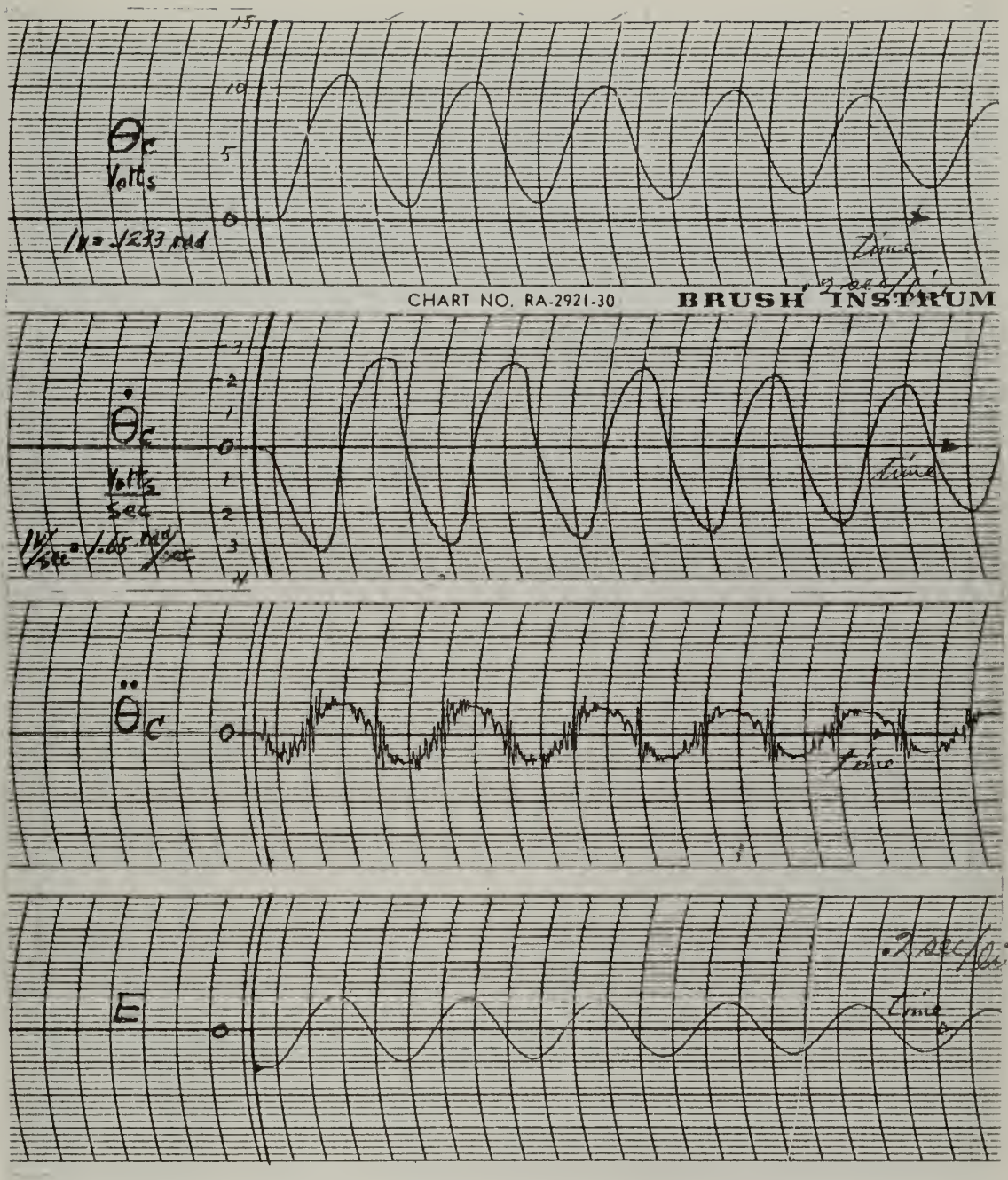


Fig. 23. Response of the physical system.

a) Underdamped.

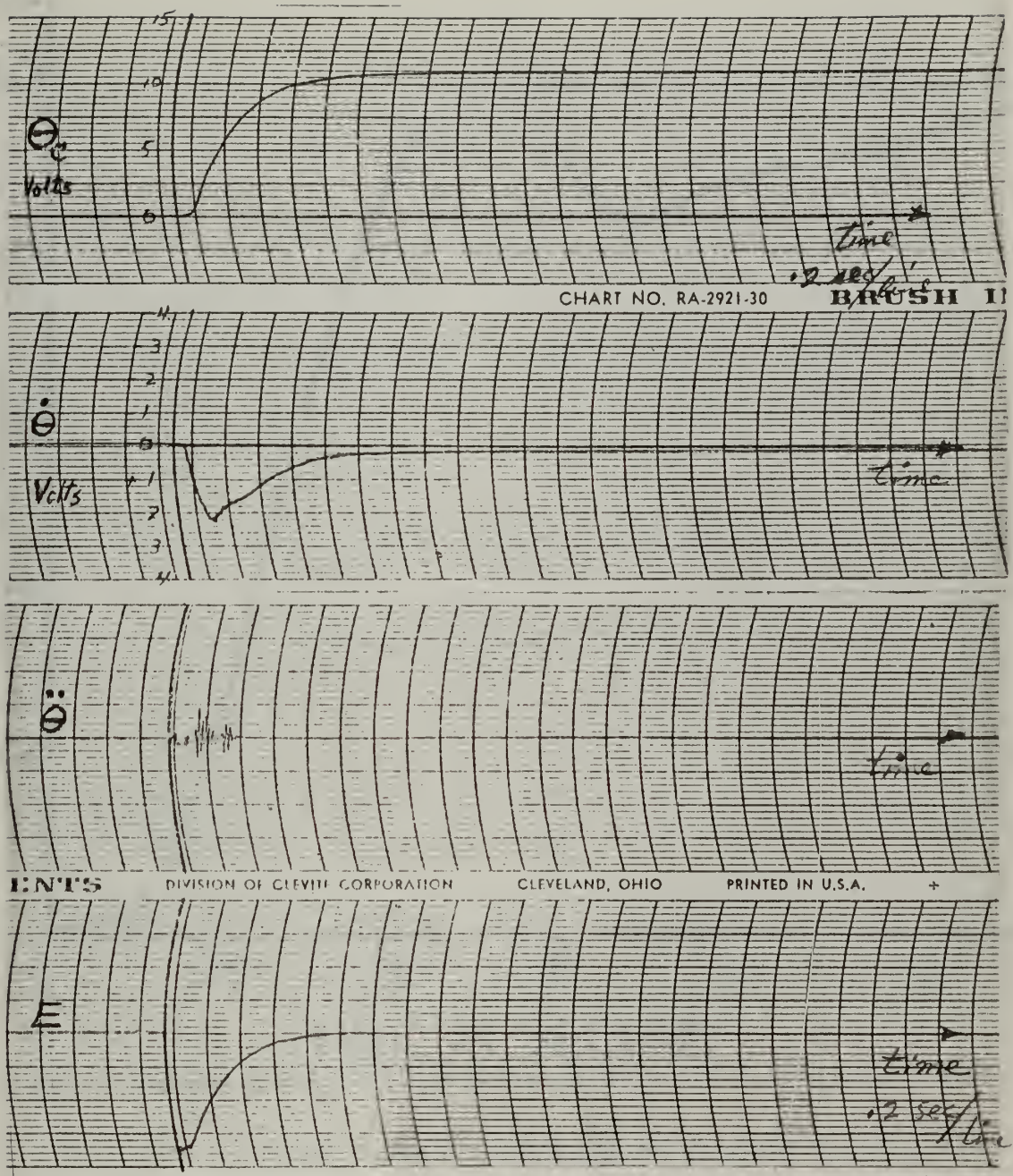


Fig. 23. Response of the physical system.

b) Overdamped.

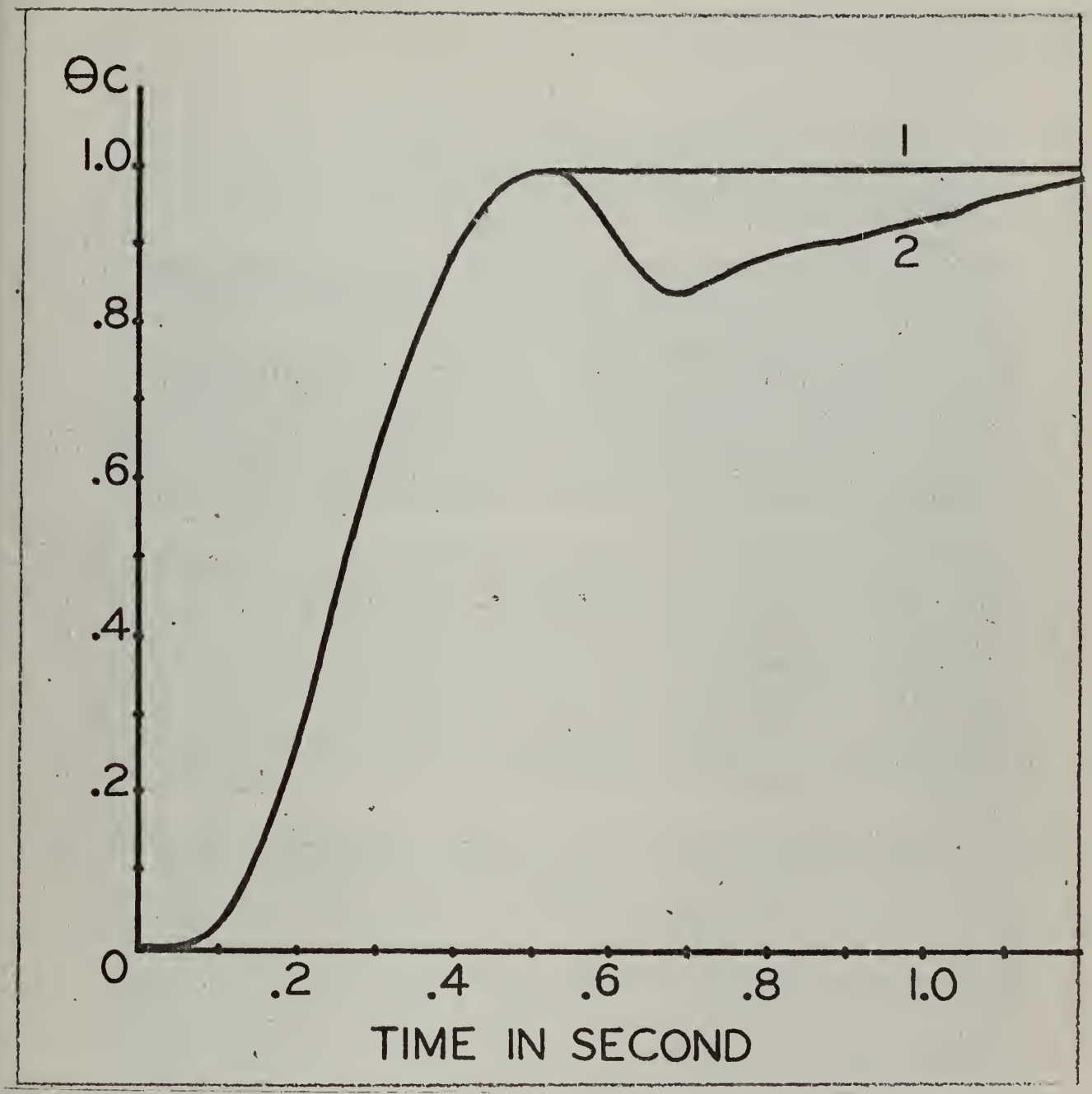


Fig. 24. Step Response of Servo with Modified Posicast Control.

Using Circuit of Fig. 22a

1) Posicast, with Switching in Quadrature Field.

2) Posicast without Cancellation of Stored Energy.

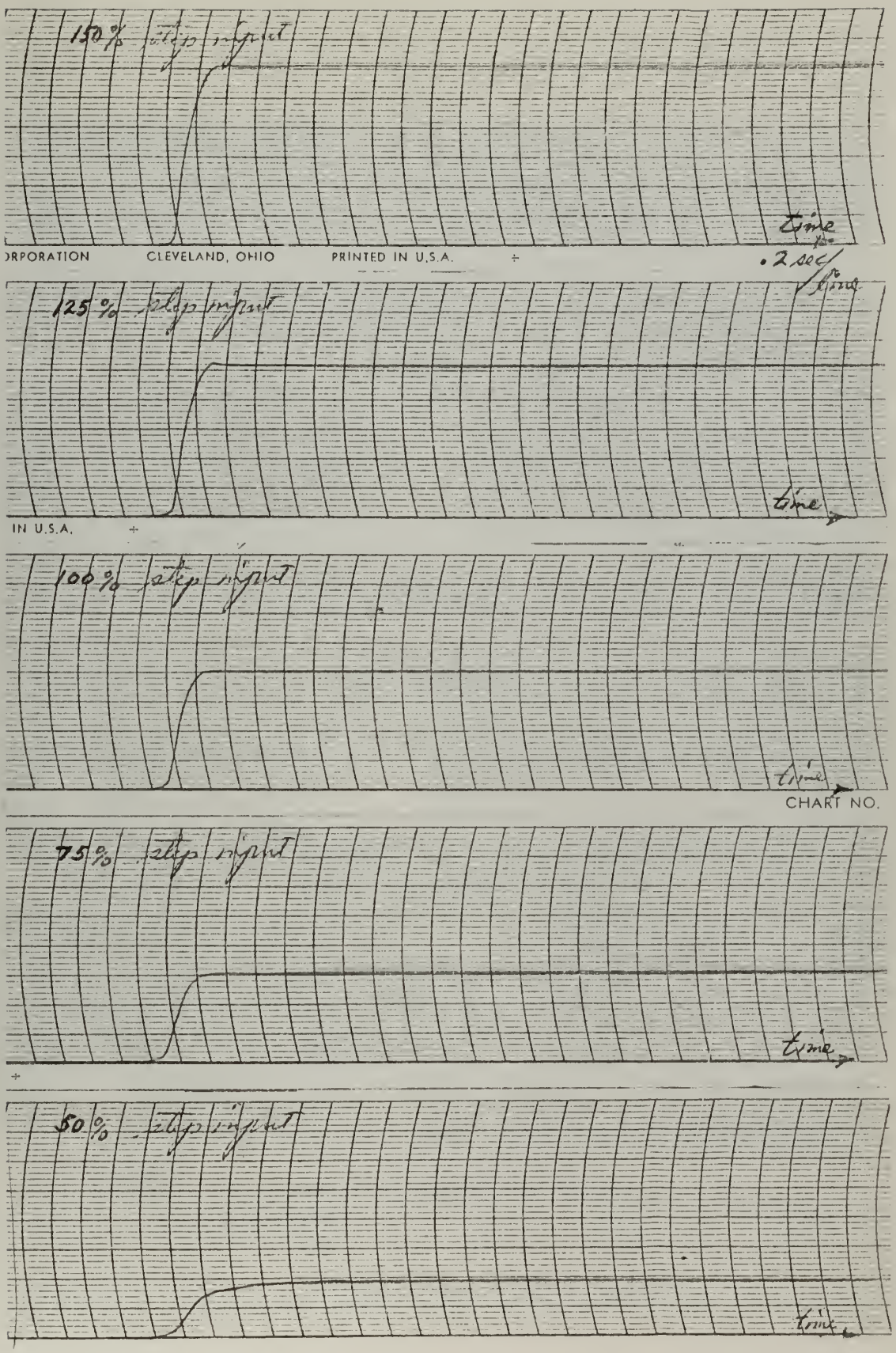


Fig. 25 (I). Family of Step Responses Opening the Quadrature Field of Amplidyne.

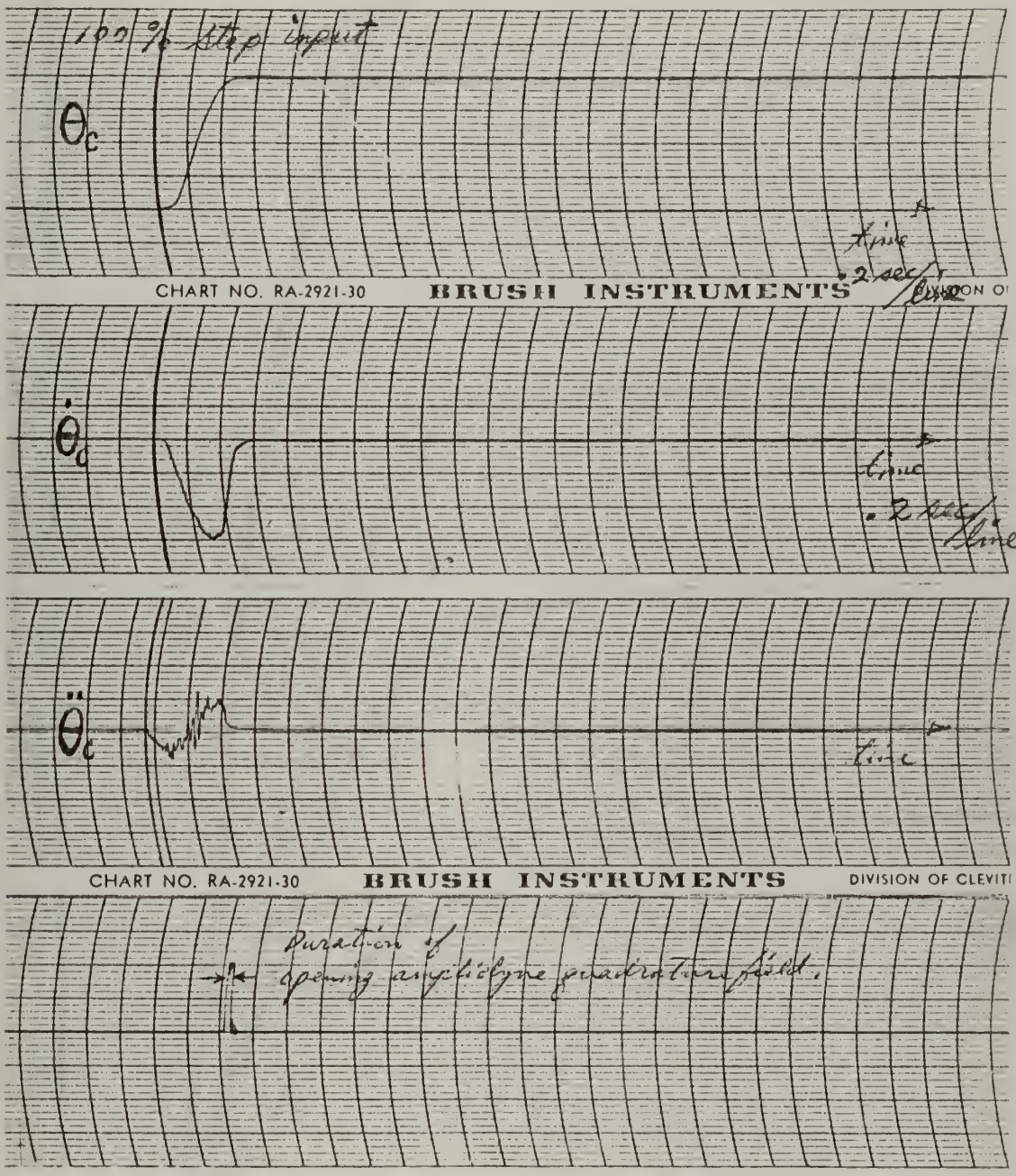


Fig. 25 (II). Step Responses Opening the Quadrature Field of Amplidyne.

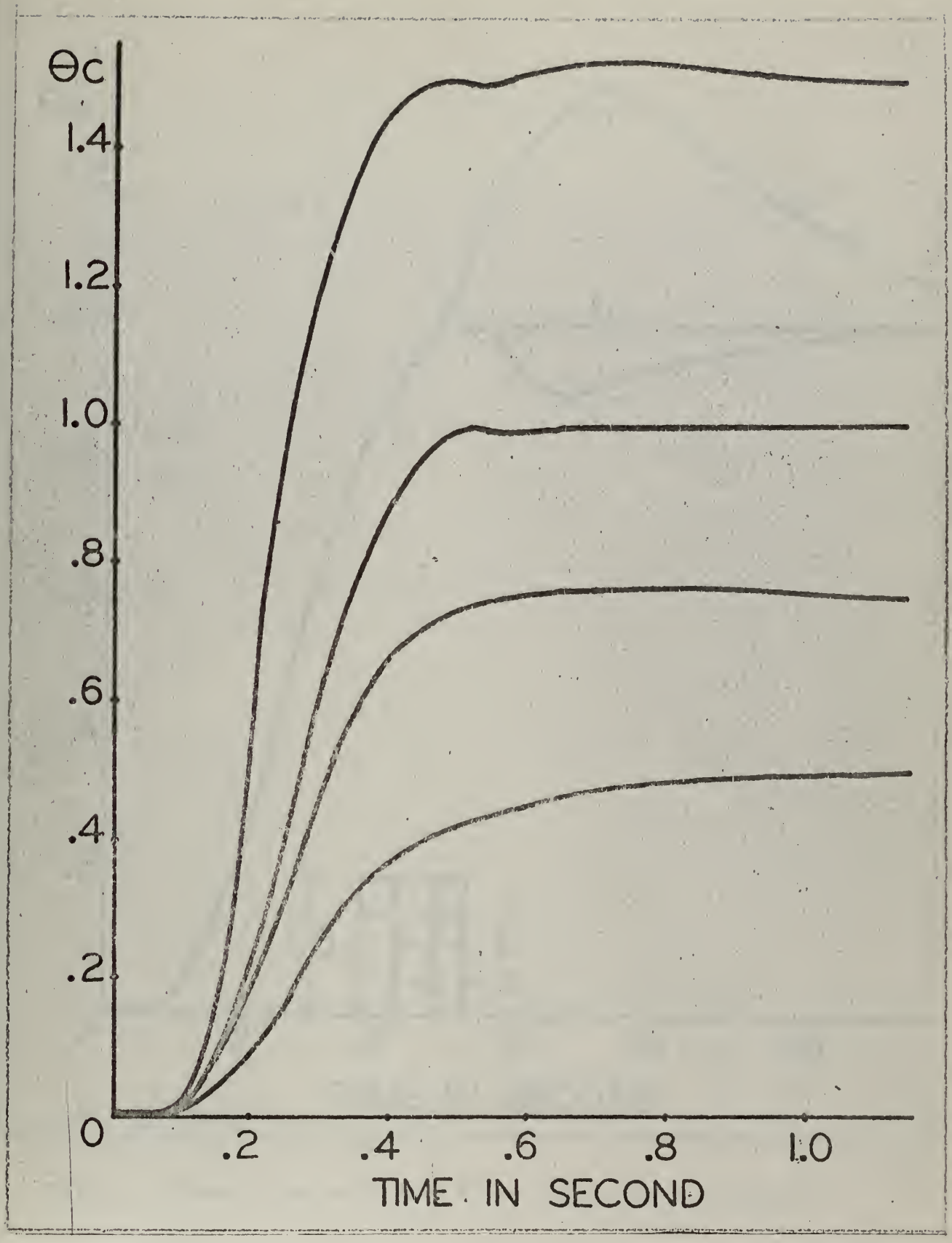
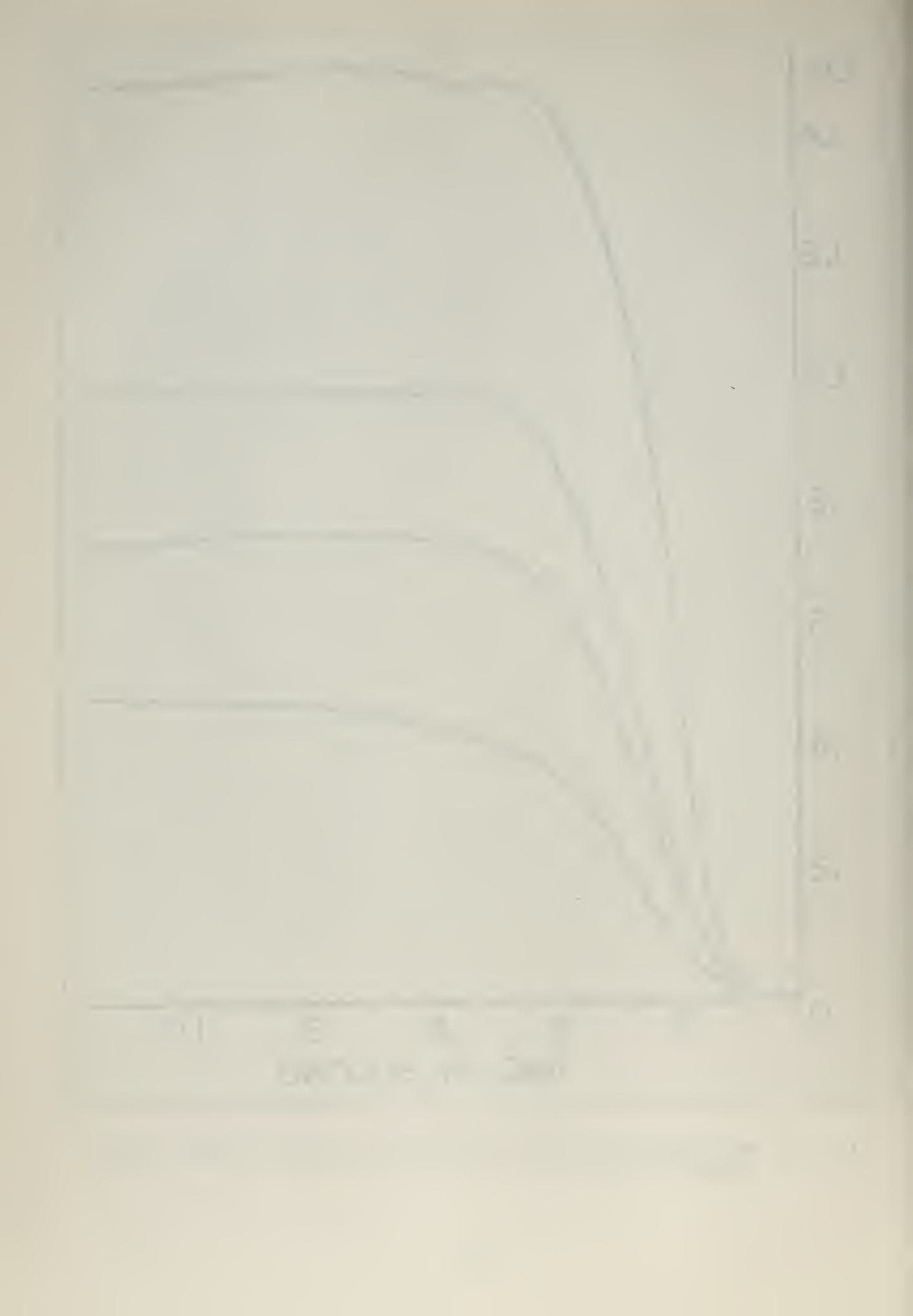


Fig. 26. Step Responses of Servo with Modified Posicast Control. Using Circuit of Fig. 22b.



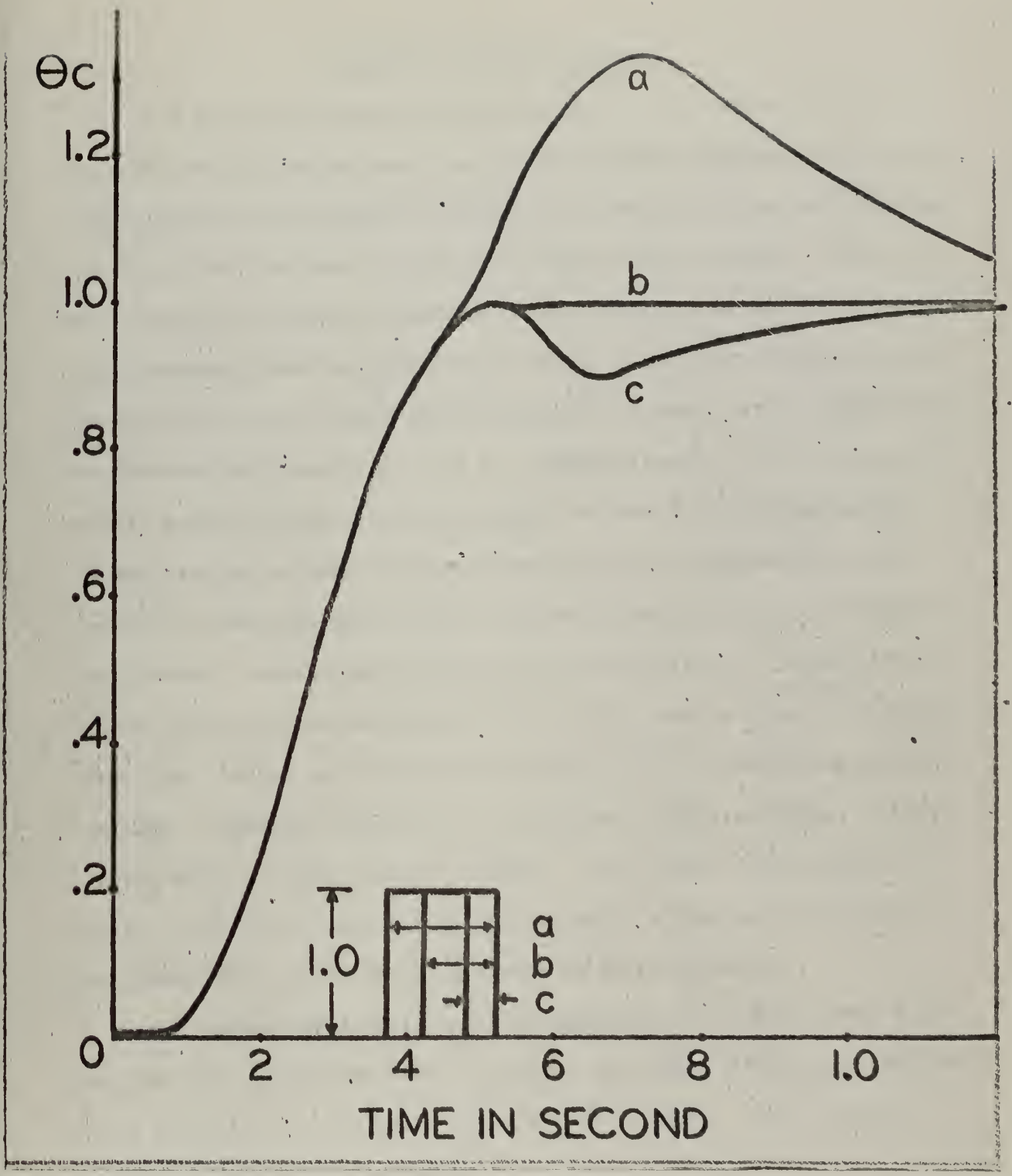
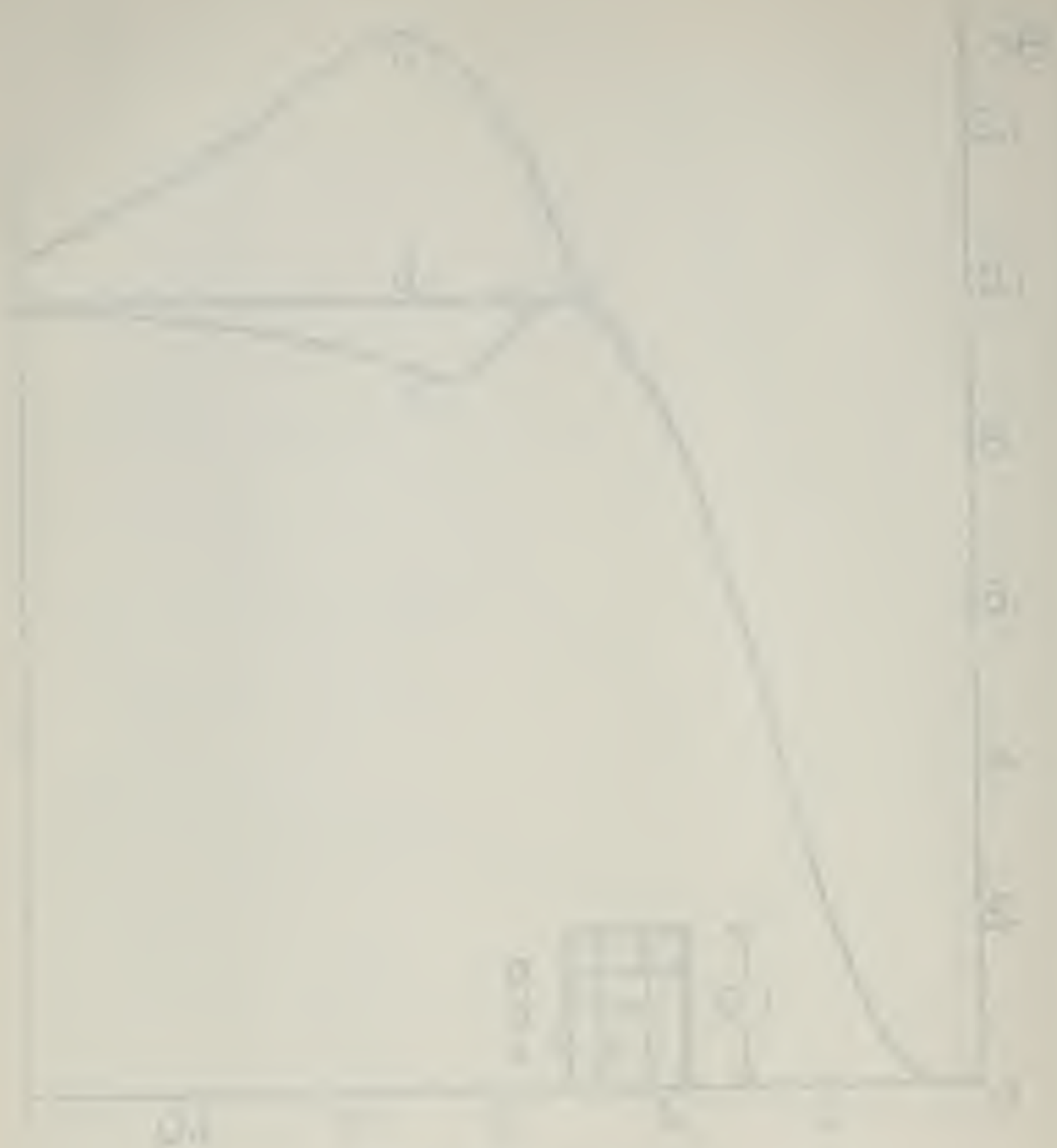


Fig. 27. Effect of Pulse Width (τ) on the Transient Response.



DISCUSSION AND CONCLUSIONS

This paper has proposed two modifications for the application of posicast control which remove some objectionable characteristics while retaining all of the desired features, and which also permit ready extension of the posicast principle to higher order systems. The first modification is that of designing damping circuits which are connected simultaneously with the application of the second part of the divided step. The purpose of this modification is to permit very light damping with consequent short rise time for command signals, while providing a stiff, heavily damped system to oppose noise and load disturbances. It has been shown that such compensation can be designed for either cascade or feedback connections, and easily satisfactorily inserted in the system. Cascade compensators are more difficult to design if the static gain is to be maintained at the level used to obtain the desired rise time, but is not difficult to obtain if this restriction may be relaxed. Feedback compensation is relatively easy to design, and the static gain is automatically retained. For higher order systems the noise level in the derivatives may introduce a practical difficulty, but such difficulties can be overcome as has been shown.

The second modification is the suggestion that simple techniques may be used to dissipate stored energy, thus permitting ready application of posicast principle to higher order systems. Three specific techniques have been suggested.

The first of these is direct dissipation of the stored energy by open or short circuit switching. This has been proven satisfactory by application to a physical servo system. The second technique is cancellation of the transient due to the stored energy by injection of a

pulse. This has also been proven satisfactory. The authors feel that dissipation of energy by switching is better than the pulse technique when both are applicable. The third technique, studied in conjunction with a simulated third order system, evolves a method for forcing E, $\dot{E}$ and $\ddot{E}$ to go to zero simultaneously. This results from a phase space study and provides a circuit which forces the system to follow a selected eigenvector in the $\dot{E}$ vs. $\ddot{E}$ plane. This technique worked satisfactorily in simulation and suggests numerous possibilities for future development.

Finally, it has been shown that both modifications may be applied simultaneously with satisfactory results.

1. Smith, O. J. M., "Posicast Control of Damped Oscillatory Systems". Proc. IRE, Vol. 45, September 1957.

2. Tallman, G. H. and O. J. M. Smith, "Analog Study of Dead-Beat Posicast Control". IRE Trans on Automatic Control, March 1958.

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